DETECTION OF CYCLOSTATIONARITY USING GENERALIZED COHERENCE

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ABSTRACT

A class of detectors for cyclostationarity is introduced. These detectors are based on the use of generalized coherence to measure correlation among two or more collections of random vectors. The generalized coherence framework allows any finite collection of pertinent samples of the cyclic auto-correlation function estimates formed from the measured signal data to be combined into the detection statistic. The performance of this approach is demonstrated and compared against other established cyclostationarity detectors in both a cognitive radio scenario and a multi-channel passive surveillance scenario.

Index Terms— Cyclostationarity, Coherence, Multiple-channel detection

I. INTRODUCTION

A zero-mean random signal $x(t)$ is said to be second-order or wide-sense cyclostationary if its auto-correlation function, $C(t,t') = \mathsf{E}\{\overline{x(t)}x(t')\}$, is periodic with some period T in the sense that $C(t+T, t'+T) = C(t,t')$ for all t and t'. The concept of cyclostationarity models the subtle periodicity exhibited by many natural and artificial processes where the signal itself is not periodic due to random variation in components of the signal, but where the statistics of the signal do exhibit periodicities [1], [2], [3]. There has been renewed interest in cyclostationary detection in the past few years in connection with cognitive radio [4], [5], [6].

In this paper, the problem of interest is that of detecting a cyclostationary signal with cycle period $T = 2\pi/\alpha$ embedded in wide-sense stationary (WSS) noise. Perhaps the most important property of cyclostationary signals is that correlation exists between the cyclostationary signal and an arbitrarily time-shifted copy of the signal modulated by the cycle frequency. For a WSS signal, such a correlation exists only at zero frequency. The two most important classes of cyclostationarity detectors both exploit this correlation, but in different ways. Detectors originating from the work of Gardner [3], [7] use a single time lag, usually zero, and consider the above correlation across one or more spatial channels to detect cyclostationary. The detectors of Dandawate and Giannakis [8] use the estimated cyclic correlation function across multiple time lags. They show that the estimates are asymptotically jointly normal, having non-zero mean in the presence of a cyclostationary signal. This asymptotic distribution is used to construct a detection statistic. Both classes of detector have been extended to the detection of cyclostationary signals in multi-channel systems [9], [7].

An alternative approach to cyclostationary detection is based on generalized likelihood ratio test (GLRT) or locally most powerful invariant test (LMPIT) formulations [10], [11], [5]. These are derived in the Fourier domain, asymptotically for large signal length. These detectors are shown in [5] to have an advantage over previous detectors for MIMO signals in which there is very significant multipath mixing. The detectors in [5] are derived under the assumption that the cyclostationary period is an integer multiple of the sampling period and that a sufficient number of independent realizations are available for processing. This means that when one tests for a candidate cycle period, first the signal needs to be re-sampled at a sample rate chosen to give an integer number of samples per cycle period and then one needs to decide how to partition the signal into "independent" realizations. The choices made are de facto choices of sampling points for the cyclostationary spectrum.

In [12] the use of the magnitude-squared coherence (MSC) estimate was proposed as a measure of spectral correlation for cyclostationary detection. It was also suggested, without further development, that the generalized coherence (GC) statistic [13], [14] might be used for poly-cyclic signal detection. The MSC estimate has a long history as a statistic for the problem of detecting common, or related, but unknown signal on two noisy channels [15]. The properties of the MSC estimate and the performance of detectors based upon it were studied extensively in the 1970s and 1980s [16], [17]. Generalized coherence, which extends the MSC concept to multiple-channel detection scenarios, received considerable attention in the 1980s and 1990s and into the early 2000s. The symmetries and invariance properties of the GC statistic were well studied during this period [18], [19], and the extent to which the GC estimate is canonical with respect to a desirable set of invariances and symmetries was examined in [14].

In this paper, the problem of detecting the presence of a cyclostationary signal of cycle period T is considered within the framework of generalized coherence detection. More specifically, a set of K time-shifted versions of the signal $x_{\tau_j}(t) = x(t+\tau_j)$ for $j = 1,\ldots,K$ is considered as one set of channels. These are then to be compared with a second set of channels consisting of the first set modulated by the cyclic frequency $x^{\alpha}_{\tau_j}(t) = x(t+\tau_j)e^{-i\alpha t}$ for $j = 1,\ldots,K$. The GC statistic is designed to detect correlation between the two sets of channels, while ignoring any internal correlation within each set. Correlation between the two sets of channels is evidence for a cyclostationary signal of cycle period T. This idea generalizes directly to multi-channel signals, to the use of multiple harmonics of the cycle frequency, and to poly-cyclic signal detection. It should be noted that statistics that have the structure of the generalized coherence statistics occur in [5]; in that work, however, coherence between data vectors is only taken over independent realizations.

The paper begins with a summary review of necessary background on cyclostationary processes and coherence detection. In Section III, cyclostationary detectors are constructed based on the GC statistic for the single-channel, multi-channel and multi-harmonic cases. In Section IV, the performance of the new detector is demonstrated in a small number of scenarios. A few closing remarks are given in Section V.

II. BACKGROUND

II-A. Cyclostationarity

A zero-mean random signal $x(t)$ is said to be second-order or wide-sense cyclostationary if its auto-correlation function, $C(t,t') = \mathsf{E}(\overline{x(t)}x(t'))$ is periodic with some period $T \neq 0$; i.e., $C(t+T, t'+T) = C(t,t')$ for all t and t'. The auto-correlation function has the property that, for each $\tau \in \mathbb{R}$,

$$C(t, t+\tau) = \sum_{n\in\mathbb{Z}} R_n(\tau) e^{in\alpha t}.$$

In this expression, $\alpha = 2\pi/T$ is the cyclic frequency and the functions

$$R_n(\tau) = \frac{1}{T}\int_{-T/2}^{T/2} C(t, t+\tau) e^{-2\pi int/T} dt$$

are the cyclic auto-correlations. The corresponding Fourier transforms of the $R_n(\tau)$ with respect to τ are the cyclic spectra.

Fundamental to almost all cyclostationary signal detection methods is the fact that

$$\mathsf{E}\left\{\frac{1}{NT}\int_{-NT/2}^{NT/2} x(t)\overline{x(t+\tau)}e^{-i\alpha t} dt\right\} = R_n(\tau).$$

This leads to the usual estimate of $R_n(\tau)$

$$\hat{R}_n(\tau) = \frac{1}{N}\langle x, x_\tau^{n\alpha}\rangle = \frac{1}{N}\sum_{k=0}^{N} x_k [x_\tau^{n\alpha}]_k. \tag{1}$$

In what follows, the shorthand notation x_τ^ω will be used for the time-shifted and frequency modulated versions of x,

$$x_\tau^\omega(t) = x(t+\tau)e^{-i\omega t},$$

and $[x_\tau^\omega]_k$ will denote k^{th} sample of this signal. Equation (1) implies that when $R_n(\tau)$ is non-zero, then a non-zero value of $\langle x, x_\tau^{n\alpha}\rangle$ can be used as evidence of the presence of a cyclostationary signal. A similar story exists in terms of spectra, but it will not be discussed here. If the time shifts are multiples of the sampling period ℓ, then

$$\hat{R}_n(\tau) = \frac{1}{N}\sum_{k=0}^{N} x_k x_{k+\ell} e^{-in\alpha k}.$$

The detection of cyclostationary signals entails two separate problems. The first is the choice of sampling points; i.e., the collections of values of (n, τ) at which estimates of $R_n(\tau)$ are made. This choice depends on the particular class of cyclostationary signals to be detected. Given any finite selection of sampling points, Karhunen-Loève analysis enables construction of a continuum of cyclostationary signals for which $R_n(\tau)$ is zero at all of the sample points. This property would make the constructed class of signals undetectable using the chosen samples, even though they are cyclostationary.

The second and separate key aspect in construction of cyclostationary detectors is how the estimates of the samples of $R_n(\tau)$ should be combined to construct a detection statistic. In detectors based on the work of Gardner et al. [3], [7], there is generally only a single sample and question of combination does not arise. The dominant method of combination in the literature is the one used by Dandawate [8], [9], whose combined statistic is based on of the asymptotic normality of the set of samples. In this paper a new and quite different approach to the construction of cyclostationary detectors is developed based on ideas from generalized coherence detection.

II-B. Generalized Coherence

Consider two sets of K channels, each digitally sampled to obtain signal vectors of length N. These two sets of K vectors in $\mathbb{C}^N$ may be regarded as the columns of two matrices X_1 and $X_2 \in \mathbb{C}^{N\times K}$. The generalized coherence between the two sets of vectors is defined as follows. First, the columns of X_1 and X_2 are orthonormalized by the transformation

$$\widetilde{X}_j = X_j(X_j^\dagger X_j)^{-1/2} \tag{2}$$

for $j = 1, 2$. Note that $\widetilde{X}_j^\dagger \widetilde{X}_j = I_K$, the $K\times K$ identity matrix. The normalized Gram matrix of the combined set of $2K$ vectors is defined in terms of the $N \times 2K$ matrix $\widetilde{\mathbf{X}} = (\widetilde{X}_1, \widetilde{X}_2)$ by

$$\widetilde{\mathbf{G}}(X_1, X_2) = \widetilde{\mathbf{X}}^\dagger\widetilde{\mathbf{X}} = \begin{pmatrix} I & \widetilde{X}_1^\dagger\widetilde{X}_2 \\ \widetilde{X}_2^\dagger\widetilde{X}_1 & I\end{pmatrix}$$

The generalized coherence is then defined in terms of $\widetilde{\mathbf{G}}$ as

$$\gamma^2(X_1, X_2) = 1 - \det\, \widetilde{\mathbf{G}}(X_1, X_2).$$

The generalized coherence is a measure of the correlation of the two sets of vectors X_1 and X_2, ignoring any correlations between sets of vectors belonging to the same set. Denoting the subspace of $\mathbb{C}^N$ spanned by the columns of X_j by $\langle X_j\rangle$ the generalized coherence is seen to be

$$\gamma^2(\mathbf{X}_1, \mathbf{X}_2) = \prod_{j=1}^{K} \cos^2\theta_j$$

where the θ_j are the principal angles between the subspaces $\langle X_1\rangle$ and $\langle X_2\rangle$.

This construction can be extended to M sets of K channels, where one is seeks to measure if there is some correlation between any or all of the M sets, irrespective of any correlations that may be present within any particular set. Write the MK signal vectors sampled from the channels as the columns of M matrices $X_1, X_2, \ldots, X_M \in \mathbb{C}^{N\times K}$. For each $m = 1, \ldots, M$, the columns of the matrix X_m are orthonormalized according to (2) to give $\widetilde{X}_m$. Writing $\widetilde{\mathbf{X}} = (\widetilde{X}_1, \ldots, \widetilde{X}_M)$ the normalized Gram matrix is

$$\widetilde{\mathbf{G}}(X_1, \ldots, X_M) = \widetilde{\mathbf{X}}^\dagger\widetilde{\mathbf{X}} = \begin{pmatrix} I & \widetilde{X}_1^\dagger\widetilde{X}_2 & \ldots & \widetilde{X}_1^\dagger\widetilde{X}_M \\ \widetilde{X}_2^\dagger\widetilde{X}_1 & I & \ldots & \widetilde{X}_2^\dagger\widetilde{X}_M \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{X}_M^\dagger\widetilde{X}_1 & \widetilde{X}_M^\dagger\widetilde{X}_2 & \ldots & I\end{pmatrix}$$

and generalized coherence is defined to be

$$\gamma^2(\mathbf{X}_1, \ldots, \mathbf{X}_M) = 1 - \det\, \widetilde{\mathbf{G}}(X_1, \ldots, X_M)$$

As a detection statistic, the generalized coherence was introduced into multi-channel signal detection as a extension of the widely used MSC estimate [13]. The motivation for its introduction was geometric. The MSC for two channels is the square of the cosine of the angle between the signal vectors from the two channels. This also one minus the square of the volume of the parallelogram formed when the two signal vectors are normalized to unit length. For M channels the general coherence, as introduced in [13], is one minus the squared volume of the parallelotope formed by the unit vectors corresponding to the M channels. When the data consists of M blocks of size K, as just discussed, the angles between vectors are replaced by angles between the K subspaces spanned by the vectors in each of the M blocks.

Under the conditions that the entries the matrix $\mathbf{X} = (X_1, \ldots, X_M)$ are zero-mean, complex normally distributed with the rows of $\mathbf{X}$ being IID, the generalized coherence can be derived

[20] as the generalized likelihood ratio for the following hypothesis test on the $MK \times MK$ covariance matrix $\mathbf{R}$ for the rows of $\mathbf{X}$:

H_1 : $\mathbf{R}$ is a general covariance matrix.

H_0 : $\mathbf{R}$ is block diagonal with $K \times K$ diagonal blocks.

In this context, the generalized coherence as a detection statistic is defined and analyzed in [21] and [22].

III. CYCLOSTATIONARY DETECTION AS GENERALIZED COHERENCE DETECTION

III-A. Single-channel detection

Begin by considering a digital signal $\boldsymbol{x}$ obtained from a single channel. As discussed above, a basic component of almost all cyclostationary signal detectors is the inner product between $\boldsymbol{x}$ and $\boldsymbol{x}_\tau^\alpha$. Since the time and frequency shift operators are unitary (or essentially unitary for sampled signals), a simple cyclostationarity detector is given by the MSC

$$\gamma^2(\boldsymbol{x}, \boldsymbol{x}_\tau^\alpha) = 1 - \frac{|\langle \boldsymbol{x}, \boldsymbol{x}_\tau^\alpha \rangle|^2}{\|\boldsymbol{x}\|^2 \|\boldsymbol{x}_\tau^\alpha\|^2}.$$

Now consider a set of K time-translated versions ("virtual channels") of the signal $\boldsymbol{x}$; i.e., $X_{\boldsymbol{\tau}} = (\boldsymbol{x}_{\tau_1}, \ldots, \boldsymbol{x}_{\tau_K})$, clipped so that the vectors are all the same length N. A second set of K channels $X_{\boldsymbol{\tau}}^\alpha = (\boldsymbol{x}_{\tau_1}^\alpha, \ldots, \boldsymbol{x}_{\tau_K}^\alpha)$ consists of the same K channels modulated by the candidate cyclostationary frequency α. Any correlations between the first set of K channels and the second is evidence of cyclostationarity, while correlation between pairs of vectors which both lie in either one of the two sets is irrelevant for assessing cyclostationarity. This suggests that cyclostationarity can be detected using the generalized coherence statistic

$$\gamma^2(X_{\boldsymbol{\tau}}, X_{\boldsymbol{\tau}}^\alpha) = 1 - \det\, \widetilde{\mathbf{G}}(X_{\boldsymbol{\tau}}, X_{\boldsymbol{\tau}}^\alpha) \tag{3}$$

where, as in Section II-B,

$$\widetilde{\mathbf{G}}(X_{\boldsymbol{\tau}}, X_{\boldsymbol{\tau}}^\alpha) = \begin{pmatrix} I & \widetilde{X}_{\boldsymbol{\tau}}^\dagger \widetilde{X}_{\boldsymbol{\tau}}^\alpha \\ \widetilde{X}_{\boldsymbol{\tau}}^{\alpha\dagger} \widetilde{X}_{\boldsymbol{\tau}} & I \end{pmatrix}.$$

Note that $\widetilde{X}_{\boldsymbol{\tau}}^\alpha$ denotes an orthonormalize after shift and modulation.

The generalized coherence statistic can also be written as

$$\gamma^2(X_{\boldsymbol{\tau}}, X_{\boldsymbol{\tau}}^\alpha) = \prod_{k=1}^{K} \cos^2 \theta_k$$

where the θ_k are the principal angles between the subspaces $\langle X_{\boldsymbol{\tau}} \rangle$ and $\langle X_{\boldsymbol{\tau}}^\alpha \rangle$

III-B. Multi-channel detection

Now consider a multi-channel system with L spatial channels. In this case, for a choice of time shifts $\tau_1, \ldots, \tau_Q$, the $K = LQ$ spatial channels and all of their time shifts are collected into $X_{\boldsymbol{\tau}}$. The matrix $X_{\boldsymbol{\tau}}^\alpha$ consists of the columns of $X_{\boldsymbol{\tau}}$ modulated by the candidate cyclostationary frequency. The generalized coherence statistic is then constructed as in (3).

III-C. Multiple Harmonics

The generalized coherence detector can also be extended to use multiple harmonics of the cyclostationary frequency. For a set of spatial and time-shifted channels $X_{\boldsymbol{\tau}}$, obtain modulated versions of $X_{\boldsymbol{\tau}}^{n_j\alpha}$ for integers $n_1, \cdots, n_H$. The multiple harmonic generalized coherence is given by

$$\gamma^2(\mathbf{X}_{\boldsymbol{\tau}}, \mathbf{X}_\tau^{n_1\alpha} \ldots, \mathbf{X}_{\boldsymbol{\tau}}^{n_H\alpha}) = 1 - \det\, \widetilde{\mathbf{G}}(\mathbf{X}_{\boldsymbol{\tau}}, \mathbf{X}_\tau^{n_1\alpha} \ldots, \mathbf{X}_{\boldsymbol{\tau}}^{n_H\alpha}) \tag{4}$$

where

$$\widetilde{\mathbf{G}} = \begin{pmatrix} I & \widetilde{X}_{\boldsymbol{\tau}}^\dagger \widetilde{X}_{\boldsymbol{\tau}}^{n_1\alpha} & \ldots & \widetilde{X}_{\boldsymbol{\tau}}^\dagger \widetilde{X}_{\boldsymbol{\tau}}^{n_H\alpha} \\ \widetilde{X}_{\boldsymbol{\tau}}^{n_1\alpha\dagger} \widetilde{X}_{\boldsymbol{\tau}} & I & \ldots & \widetilde{X}_{\boldsymbol{\tau}}^{n_1\alpha\dagger} \widetilde{X}_{\boldsymbol{\tau}}^{n_H\alpha} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{X}_{\boldsymbol{\tau}}^{n_H\alpha\dagger} \widetilde{X}_{\boldsymbol{\tau}} & \widetilde{X}_{\boldsymbol{\tau}}^{n_H\alpha\dagger} \widetilde{X}_{\boldsymbol{\tau}}^{n_1\alpha} & \ldots & I \end{pmatrix}.$$

The coherence detector here detects any correlation across any of the sets $X_{\boldsymbol{\tau}}, X_{\boldsymbol{\tau}}^{n_1\alpha}, \ldots, X_{\boldsymbol{\tau}}^{n_H\alpha}$.

IV. PERFORMANCE

This section provides an evaluation of the detection performance of the proposed detector (4), labeled as "GC" in the figures. Its performance is compared with the main existing techniques; i.e., those given in [5], [9] and [7]. In the figures, these are labeled as "Ramirez et al", "Lunden et al" and "Shell&Gardner", respectively. Two scenarios are considered in this section. The first is a cognitive radio application which tests whether the received signal is cyclostationary (CS) versus wide-sense stationary (WSS). The second example is a multi-channel passive detection application where the test is CS against Gaussian white noise (GWN). In both cases, the signal has a QPSK modulation and, for fair comparison, the total number of symbols use for all detector is the same. The detector from Ramírez et al. [5] is implemented by cutting the signal into 15 snapshots each containing 256 symbols at each antenna.

IV-A. Example 1

This example considers a cognitive radio scenario as given by Ramírez et al. in [5]. The received signal $\boldsymbol{x}(n) \in \mathbb{C}^M$ observed by M sensors is given by

$$\boldsymbol{x}(n) = (\mathbf{H} * \boldsymbol{s})(n) + \boldsymbol{w}(n)$$

where $\boldsymbol{w}(n) \in \mathbb{C}^M$ is additive correlated Gaussian noise. The noise is generated by applying a moving-average filter of order 19.

The signal $\boldsymbol{s}(n) \in \mathbb{C}^M$ is a QPSK signal with rectangular shaping and a symbol rate of 300 Kbaud. The MIMO channel matrix $\mathbf{H}(n) \in \mathbb{C}^{M\times M}$ is a Rayleigh channel without antenna correlation and has an exponential power delay profile, with a maximum delay of $24\,\mu s$, and a delay spread of $6.24\,\mu s$. The sampling frequency is 1.2 MHz, which gives the cycle period of 4 samples. The channel and noise coefficients are Gaussian and randomly generated in each Monte Carlo simulation. Note that the detector described in [5] requires multiple realizations, which are obtained by cutting the signal into 15 segments.

Fig. 1 shows the receiver operating characteristic (ROC) curves for the scenario with SNR $=-16$ dB and $M = 3$ sensors. The number of lags (τ) or delays used in the GC detector and the detector from [9] is 8, which is two cycles of the cyclostationary period under test. Only the fundamental harmonic is used by the GC detector. As for the detector in [7], the technique conventionally uses zero lag ($\tau = 0$). However, for a complex phase coded signal, $|x(t)|^2$ is a constant and the detector is sampling in a place where the cyclic auto-correlation is zero. This is remedied by using a lag of 1 sample. As shown in the figure, the GC detector outperforms the other detectors in this scenario.

Fig. 2 shows the results for the same scenario, but with the coloured Gaussian noise replaced by white Gaussian noise (WGN) of the same power. The performance of the detector in [5] is, somewhat puzzlingly, much degraded with this change.

IV-B. Example 2

This example involves a signal from a multi-channel passive surveillance system with an array of M receivers. The received signal $\boldsymbol{x}(n) \in \mathbb{C}^M$, observed by the M sensors is modeled as

$$\boldsymbol{x}(n) = \mathbf{a}(\theta)\boldsymbol{s}(n) + \boldsymbol{\nu}(n)$$

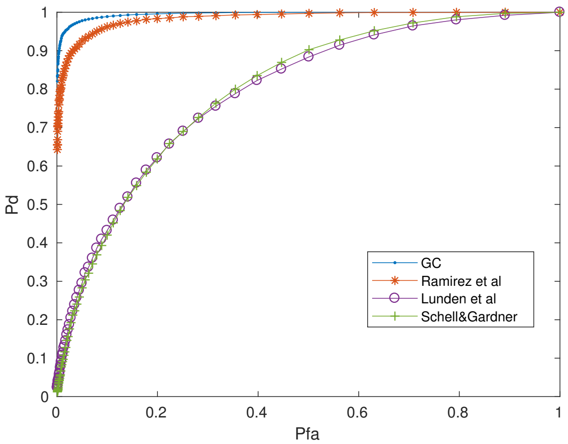

Fig. 1. ROC curves for CS vs. WSS, $M = 3$, SNR = -16dB

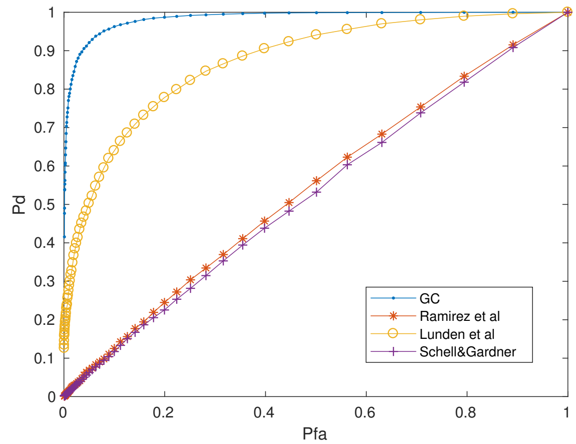

Fig. 3. ROC curves for CS vs. WGN, $M = 4$, SNR = -20dB

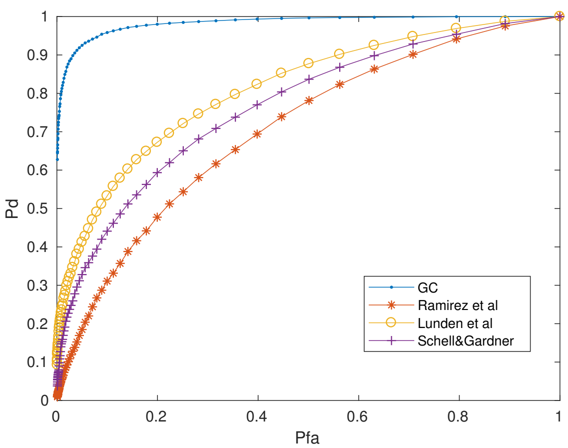

Fig. 2. ROC curves for CS vs. WGN, $M = 3$, SNR = -16dB

where $\mathbf{a}(\theta)$ is the steering vector in the source direction θ. The signal $\boldsymbol{s}(n) \in \mathbb{C}^M$ is a QPSK modulated signal with rectangular shaping and a symbol period of 50 ns. It is assumed that the receiver has 133 MHz bandwidth and its centre frequency is set at 9.4 GHz. The data is digitally sampled at $8/3 \times 10^8$ samples per second. This gives a cyclic frequency for the QPSK signal of 13.3333 samples/cycle. To apply the detector in [5], the data is re-sampled by a factor $6/5$ to give 14 samples/cycle. The receiver noise $\boldsymbol{\nu}(n)$ is taken to be distributed as $\mathcal{CN}(0, \sigma^2 I_M)$ and assumed to be independent across the sensors.

Fig. 3 shows ROC curves for this scenario with SNR$= -20$ dB and the receive array consisting of $M = 4$ elements which are spaced at a half wavelength for 18 GHz. In this example, 13 lags are used in the GC and "Lunden et al" [9] detectors, which is approximately one cycle for the cyclic period under test. As in Example 1, a time lag of 1 sample is used for the detector in [7]. As shown, in this scenario, the proposed detector outperforms all other detectors. The detectors of Ramírez et al. [5] and Schell and Gardner [7] perform rather poorly in this scenario at this level of SNR.

V. DISCUSSION AND CONCLUSIONS

Periodic correlation in the auto-correlation function of a zero-mean process can manifest in the cyclic spectra $R_n(\tau)$ for every pair (n, τ). But, as noted in Section II-A, given any finite set of sample points in n and τ, it is possible to construct a wide-sense cyclostationary signal for which $R_n(\tau)$ is zero at every sample point in the set. Established cyclostationarity detectors base their decisions on statistics that exploit a small number of fixed sample points. They are not generic cyclostationarity detectors in the sense that there are many cyclostationary signals that their design is not suited to detect.

The class of detectors introduced in this paper, while also not generic, improve this situation in two respects. First, the generalized coherence framework will naturally accommodate incorporating numerous sample points into the detection statistic, thus broadening the sets of signals they can be designed to detect. Second, they can capitalize on situations in which multiple sample points evince cyclostationarity, which should improve detector performance. Indeed, the results given in Section IV suggest that this structure offers significant possibilities for performance improvement over established methods. Note also that the ability to design detectors that exploit particular sets of sample points opens the possibility of customizing detectors to target signals with particular modulation types.

Ongoing work is investigating statistical characterization of the generalized coherence based detection statistics presented in this paper. In particular, it would be highly desirable to obtain analytical expressions for their probability distributions under "null hypotheses" that model non-cyclostationary signals – ideally including both white and WSS noise.

ACKNOWLEDGEMENT

The authors would like to thank Dr. David Ramírez of Signal Processing Group, University Carlos III of Madrid, Leganés, Spain for a fruitful discussion and sharing code for comparison.

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