COMPUTING THE LARGEST EIGENVALUE DISTRIBUTION FOR COMPLEX WISHART MATRICES

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***Abstract*—In multi-channel detection, sufficient statistics for Generalized Likelihood Ratio and Bayesian tests are often functions of the eigenvalues of the Gram matrix formed from data vectors collected at the sensors. When the null hypothesis is that the channels contain only independent complex white Gaussian noise, the distributions of these statistics arise from the joint distribution of the eigenvalues of a complex Wishart matrix G. This paper considers the particular case of the largest eigenvalue λ_1 of G, which arises in passive radar detection of a rank-one signal. Although the distribution of λ_1 is known analytically, calculating its values numerically has been observed to present formidable difficulties. This is particularly true when the dimension of the data vectors is large, as is common in passive radar applications, making computation of accurate detection thresholds intractable. This paper presents results that significantly advance the state of the art for this problem.**

***Index Terms*—Wishart matrix, Multi-channel detection, Passive radar, CFAR thresholds**

I. Introduction

Complex Wishart matrices are well-studied in the statistical literature (e.g., [1]–[3]), and they arise naturally in multi-sensor statistical signal processing applications where the received data are modeled as complex normal vectors [4]–[9] and in MIMO communications (e.g., [10]–[13]). In such applications, statistics used for signal detection and characterization (e.g., rank estimation) are often functions of the eigenvalues of the Gram matrix formed from the data vectors, which is a complex Wishart matrix under typical assumptions. Of particular recent interest is the problem of detection in multistatic passive radar. In this application, the presence of a common signal on several noisy channels of sensor data is to be ascertained, but the signal of interest is not known. Often one or more "reference channels" are available which are known to contain the signal, but this need not be the case; i.e., all channels may be "surveillance channels" which may or may not contain the signal. This latter scenario is discussed in [14], [15], and it is the primary motivation for the work presented in this paper. The results apply to a broader class of detection and estimation problems, including situations involving one reference channel owing to invariance properties of the joint distribution of the eigenvalues of the Gram matrix [16], [17].

In [6], the largest eigenvalue λ_1 of the $M \times M$ Gram matrix formed from the complex sensor data is shown to be a sufficient statistic for the Generalized Likelihood Ratio Test (GLRT) for a rank-one signal in M channels of additive complex zero-mean white Gaussian noise (ZMWGN). It is noted in [18] that under the null hypothesis where no signal component is present and the data are independent complex ZMWGN vectors, λ_1 follows the distribution of the largest eigenvalue of a $M \times M$ complex Wishart matrix. It is emphasized in [15] that, although this distribution is known, it is not straightforward to compute – especially when the dimension N of the data vectors is large. In typical passive radar scenarios, the number of receive antennas is usually small and long integration times are demanded by the very low signal-to-noise ratios commonly involved when collecting scattered signals from opportunistic sources. Thus setting thresholds that correspond to desirably low false-alarm probabilities in passive radar entails calculating the value of the cumulative distribution function (CDF) of λ_1 for small values of M large values of N. The Tracy-Widom distribution [19], [20], which is asymptotic in both N and M, has been observed to yield inaccurate detection thresholds when used to approximate the CDF of λ_1 in this regime. Despite progress reported in [15], the methods given there are feasible only up to $N \approx 4 \times 10^4$ for problems with more than three sensors whereas practical applications often involve values of N on the order of $10^5 - 10^6$ or larger.

This paper introduces an approach for computing the distribution of λ_1 that is effective for practical ranges of parameters found in passive radar applications. Section II summarizes briefly the physical model of the passive radar scenario that is the primary motivation for this work and gives the corresponding data model. In section III, detection in this scenario is discussed and role of λ_1 as a detection statistic is explained. This section also presents known expressions for the null distribution of λ_1 in terms of gamma functions and gives an overview of the numerical issues with this direct implementation of the expressions. The current state of the art in mitigating these issues, as presented in [15], is also summarized. In section VI, the relationship between Laguerre and Hermite polynomials is discussed and an equivalent set of polynomials orthogonal over the region of interest is constructed via change of variables. This mathematical machinery is used in section VII to construct expressions for the elements of the distribution matrix in terms of inner products of orthogonal polynomials. The expressions given will not overwhelm floating point representations, and the integrals can be computed using numerical quadrature.

II. Signal Model

This paper considers the passive radar scenario described in [15]. The positions of the transmitter and M receivers are assumed known and the presence or absence of a target with a given isolated state (position and velocity) is to be deduced from the sensor data. As in [15], it is assumed that the transmitted signal only manifests in the data through scattering off the target; i.e., no direct-path signal appears in the sensor data, and there is no clutter. In practice, this situation would occur if physical obstacles prevent direct-path propagation of the transmitted signal to the receivers. Although motivated by this scenario, the results presented below are more broadly applicable in multi-channel detection.

In addition to a scalar gain and additive Gaussian noise, each channel imparts time delay and Doppler to the transmitted signal in accordance with the putative target state and the corresponding sensor position. Delay and Doppler are compensated at each receiver to obtain M complex data vectors, each of length N, which are tested for common signal content.

Under the null hypothesis $\mathcal{H}_0$ that the received data contain only noise, these vectors $\tilde{x}_m$, $m = 1, \ldots, M$ are given by

$$\mathcal{H}_0 : \tilde{x}_m = \xi_m$$

where the ξ_m are independent N-vectors of zero-mean complex Gaussian noise. Under the alternative hypothesis,

$$\mathcal{H}_1 : \tilde{x}_m = \mu_m s + \xi_m$$

where μ_m is a complex channel gain and s is a complex N-vector representing the common signal component across all M channels.

III. Numerical Issues

Considering $\mathcal{H}_0$, suppose X is an $M \times N$ matrix with independent identically distributed complex normal entries having mean zero and unit variance; i.e., $x_{ij} \sim \mathcal{CN}(0,1)$. The Gram matrix $G = XX^\dagger$ generated from this data has a central complex Wishart distribution $G \sim \mathcal{CW}(N, I_M)$. As noted above, the GLRT for detection of a rank-one signal in this setting is based on the largest eigenvalue λ_1 of this Gram matrix, and hence establishing thresholds for constant false-alarm rate detection requires explicit evaluation of the CDF of this eigenvalue. The CDF of λ_1, as given in [10], is

$$F_N(x) = P(\lambda_1 \le x) = \frac{|\gamma(N+i+j, x)|_{i,j=0,\ldots,M-1}}{|\Gamma(N+i+j)|_{i,j=0,\ldots,M-1}} \quad (1)$$

where $\gamma(a,x) = \int_0^x t^{a-1} e^{-t} dt$ is the lower incomplete gamma function.

The form of the CDF F_N presented in equation (1) strongly constrains the problem size for which explicit calculations are possible using floating point arithmetic. Direct computation using a naive implementation of this expression, while exact in principle, is severely limited by the maximum number of samples possible before overflowing double precision floating point (see Table I).

TABLE I
Computational limits of expression from [10] in double precision floating point arithmetic

Sensors (M)	Maximum Samples (N)
2	98
3	71
4	57
5	47

Realistic problems for passive radar applications require numbers of samples N on the order of $10^5 - 10^6$, generally with a single-digit number of receivers. The current best implementation, given in [15], allows for computation in problems of this size only for two or three receivers (see Table II).

TABLE II
Computational limits of expression from [15] in double precision floating point arithmetic

Sensors (M)	Maximum Samples (N)
2	4.8×10^8
3	2.6×10^5
4	4.5×10^3
5	5.5×10^2

IV. Laguerre Polynomial Expansion

This section introduces an approach to mitigate the issues noted in the preceding section. Beginning with equation (1), the CDF of λ_1 may be expanded using the generalized Laguerre polynomials. First, let A be the lower triangular matrix of coefficients of the normalized generalized Laguerre polynomials with coefficient $a = N - 1$ [21, ch. 22]. Note that these polynomials are orthonormal with respect to the measure $t^a e^{-t} dt$. Begin by pre- and post-multiplying the matrices in (1) by A:

$$\begin{aligned} F_N(x) &= \frac{|\gamma(N+i+j, x)|_{i,j=0,\ldots,M-1}}{|\Gamma(N+i+j)|_{i,j=0,\ldots,M-1}} \\ &= \frac{\left|A\left[\int_0^x t^i t^j t^a e^{-t} dt\right] A^\mathsf{T}\right|}{\left|A\left[\int_0^\infty t^i t^j t^a e^{-t} dt\right] A^\mathsf{T}\right|} \\ &= \frac{\left|\sqrt{\frac{i!j!}{(a+i)!(a+j)!}} \int_0^x L_i^{(a)}(t) L_j^{(a)}(t) t^a e^{-t} dt\right|}{\left|\sqrt{\frac{i!j!}{(a+i)!(a+j)!}} \int_0^\infty L_i^{(a)}(t) L_j^{(a)}(t) t^a e^{-t} dt\right|}. \end{aligned}$$

The denominator evaluates to δ_{ij} from the definition of the generalized Laguerre polynomials [21]. Thus, $F_N(x) = |\Psi^{(a)}|$ where the elements of the matrix $\Psi^{(a)}$ are given by

$$\Psi_{ij}^{(a)} = \sqrt{\frac{i!j!}{(a+i)!(a+j)!}} \int_0^x L_i^{(a)}(t) L_j^{(a)}(t) t^a e^{-t} dt.$$

The change of variable $t \to a + x\sqrt{2a}$ yields

$$\Psi_{ij}^{(a)}(a + x\sqrt{2a}) = \sqrt{\frac{i!j!}{(a+i)!(a+j)!}} \times$$

$$\int_{-\sqrt{\frac{a}{2}}}^{x} L_i^{(a)}(a+t\sqrt{2a})L_j^{(a)}(a+t\sqrt{2a})(a+t\sqrt{2a})^a e^{-(a+t\sqrt{2a})}dt$$

$$= \sqrt{2a}a^a e^{-a}\sqrt{\frac{i!j!}{(a+i)!(a+j)!}} \times$$

$$\int_{-\sqrt{\frac{a}{2}}}^{x} L_i^{(a)}(a+t\sqrt{2a})L_j^{(a)}(a+t\sqrt{2a})(1+t\sqrt{2/a})^a e^{-t\sqrt{2a}}dt. \quad (2)$$

V. Remarks on Computing the Matrix Elements

Note that the expression $F_N(x) = |\Phi^{(a)}|$ is exact, but computing the matrix elements $\Phi_{ij}^{(a)}$ still presents difficulties. In particular, the factor multiplying the integral in equation (2) contains factorials of the number of samples. This section develops a numerically tractable approximation for (2) that becomes arbitrarily accurate as more terms of a convergent series are used.

Applying Stirling's approximation yields

$$\sqrt{2a}a^a e^{-a}\sqrt{\frac{i!j!}{(a+i)!(a+j)!}}$$

$$= \frac{\sqrt{2a}a^a e^{-a}}{a!a^{i/2}a^{j/2}\sqrt{\Pi_{k=1}^{i}(1+k/a)\Pi_{k=1}^{j}(1+k/a)}}$$

$$= \frac{1}{\sqrt{\pi}}a^{-i/2}a^{-j/2}\frac{e^{-\epsilon(a)}}{\sqrt{\Pi_{k=1}^{i}(1+k/a)\Pi_{k=1}^{j}(1+k/a)}}$$

where the function ϵ is defined by

$$\epsilon(a) = \log a! - a\log a + a - \frac{1}{2}\log 2\pi a$$

$$= \frac{1}{12a} - \frac{1}{360a^3} + \frac{1}{1260a^5} - \frac{1}{1680a^7} + \dots.$$

Denote

$$c_{ij}(a) = \frac{e^{-\epsilon(a)}}{\sqrt{\Pi_{k=1}^{i}(1+k/a)\Pi_{k=1}^{j}(1+k/a)}}$$

and observe that $\lim_{a\to\infty} c_{ij}(a) = 1$. Next, define

$$\phi_a(t) = a\log(1+t\sqrt{2/a}) - t\sqrt{2a}.$$

For $|t| < \sqrt{a/2}$, this can be expanded in a Taylor series as

$$\phi_a(t) = -t^2 - \sum_{j=3}^{\infty}\frac{(-1)^j}{j}(t\sqrt{2/a})^j.$$

Finally, substituting c_{ij} and ϕ_a into the integral in (2) produces

$$\Psi_{ij}^{(a)}(a+\sqrt{2a}x) = c_{ij}(a)\frac{\sqrt{i!j!}}{\sqrt{\pi}} \times$$

$$\int_{-\sqrt{a/2}}^{x} a^{-i/2}L_i^{(a)}(a+t\sqrt{2a})a^{-j/2}L_j^{(a)}(a+t\sqrt{2a})e^{\phi_a(t)}dt. \quad (3)$$

VI. Asymptotics of Generalized Laguerre Polynomials

Consider the relation between the generalized Laguerre and Hermite polynomials shown in [22]:

$$\lim_{a\to\infty} a^{-n/2}L_n^{(a)}(a+t\sqrt{a}) = \frac{(-1)^n}{n!}2^{-n/2}H_n(t/\sqrt{2}).$$

Define polynomials

$$D_n^{(a)}(t) = (-1)^n n!(2/a)^{n/2}L_n^{(a)}(a+t\sqrt{2a}).$$

As a consequence of the Laguerre recurrence relation

$$L_{n+1}^{(a)}(x) = \frac{(2n+1+a-x)}{n+1}L_n^{(a)}(x) - \frac{(n+a)}{n+1}L_{n-1}^{(a)}(x),$$

the polynomials $D_n^{(a)}$ satisfy the recurrence relation

$$D_{n+1}^{(a)}(x) = \left(2x - (2n+1)\sqrt{2/a}\right)D_n^{(a)}(x) - \left(2n + n^2(2/a)\right)D_{n-1}^{(a)}(x)$$

$$D_0^{(a)}(x) = 1 \qquad D_1^{(a)}(x) = 2x - \sqrt{2/a}.$$

Taking the limit $a \to \infty$, this recurrence relation is the same as that satisfied by the Hermite polynomials, with the same initial conditions. Therefore

$$\lim_{a\to\infty} D_n^{(a)} = H_n. \quad (4)$$

These polynomials are orthogonal on the interval $[-\sqrt{a/2}, \infty)$ with respect to the measure $e^{\phi_a(t)}dt$; i.e.,

$$\int_{-\sqrt{a/2}}^{\infty} D_n^{(a)}(t)D_m^{(a)}(t)e^{\phi_a(t)}dt = \frac{\sqrt{\pi}2^n n!}{c_{nn}(a)}\delta_{nm}.$$

VII. The Distribution

In order to write F_N in a form that can be computed using floating point arithmetic for practical problem sizes, the results on orthogonal polynomials in Section VI are combined with the integral forms shown in Sections IV and V.

A. Hermite

As $a \to \infty$ in equation (4), $D_n^{(a)} \to H_n$. Therefore, the distribution of λ_1 can be written in terms of the Hermite polynomials as

$$F_{a+1}(a+\sqrt{2a}x) = \left|\frac{1}{\sqrt{\pi 2^{i+j}i!j!}}\int_{-\infty}^{x} H_i(t)H_j(t)e^{-t^2}dt\right|. \quad (5)$$

With some additional recurrence relations, the elements of this matrix can be computed without computing any integrals. First, consider that $d(H_j(t)e^{-t^2}) = -H_{j+1}(t)e^{-t^2}$. For the zeroth row of the matrix, the inner product takes the form

$$\int_{-\infty}^{x} H_i(t)H_j(t)e^{-t^2}dt = \begin{cases} \sqrt{\pi}(1+\mathrm{erf}(x))/2 & \text{if } j = 0 \\ -H_{j-1}(x)e^{-x^2} & \text{otherwise.} \end{cases}$$

Subsequent elements of the matrix can be recursively calculated using the relation

$$\int_{-\infty}^{x} H_i(t)H_j(t)e^{-t^2}dt = -H_i(x)H_{j-1}(x)e^{-x^2} + 2i\int_{-\infty}^{x} H_{i-1}(t)H_{j-1}e^{-t^2}dt.$$

The formulation of the distribution shown in equation (5) eliminates the large gamma functions with number of degrees of freedom (samples), thus eliminating the main cause of the severe floating point overflow.

B. D Polynomials

Substituting the D polynomials defined in Section VI into the integral form of the matrix elements shown in (3) gives the matrix elements in terms of partial inner products of the D polynomials:

$$\Psi_{ij}^{(a)}(a+\sqrt{2a}x) = \frac{c_{ij}(a)}{\sqrt{\pi 2^{i+j} i! j!}} \times \int_{-\sqrt{a/2}}^{x} D_i^{(a)}(t) D_j^{(a)}(t) e^{\phi_a(t)} dt.$$

The distribution can now be computed by taking the determinant of this matrix:

$$F_{a+1}(a+\sqrt{2a}x) = \left| \frac{c_{ij}(a)}{\sqrt{\pi 2^{i+j} i! j!}} \int_{-\sqrt{a/2}}^{x} D_i(t) D_j(t) e^{\phi_a(t)} dt \right|. \tag{6}$$

As seen in the previous subsection, the only factorial or gamma function terms have arguments in terms of the number of receiver channels, which for passive radar applications is generally a single-digit number, resulting in a leading term easily managed by floating point representations.

VIII. Numerical Results

This section describes the results of numerical computations of the CDF for λ_1 for $\mathcal{H}_0$ using three methods. Direct Monte Carlo simulations of complex Wishart matrices will be used as a baseline, and will be compared to the methods constructed in the preceding sections of this paper, shown in equations (5) and (6).

A. Complementary CDF Comparison

Shown here are figures demonstrating the complementary CDF with $M = 2$ sensors and $N = 10^4$, the approximate limit of the method demonstrated in [15], as well as for $M = 5$ and $N = 10^6$, demonstrating the viability of the new methods to practical passive radar problems. Monte Carlo simulations with one million trials at each point were performed.

The new methods agree with Monte Carlo simulation up to the number of trials; larger experiments would be too computationally time intensive. In addition, note the convergence of tail probabilities for larger values of N in the curves for equation (5).

B. Floating Point Limits

Similar to Section III, empirical testing was performed to find general limits for the number of degrees of freedom before equation (6) overwhelms double precision floating point. In tested cases with large numbers of samples, limits to the number of bins in built in numerical integration algorithms begin to cause accuracy issues when the number of samples exceeds numbers on the order of 10^9. Taking more care with the choice of integration algorithm and parameters would likely allow this threshold to be exceeded.

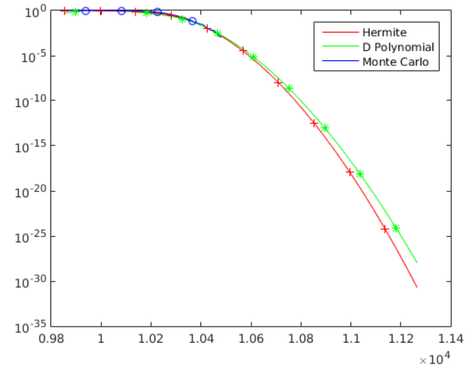

Fig. 1. Complementary CDF comparison from equations (5), (6) for $M = 2$ and $N = 10^4$.

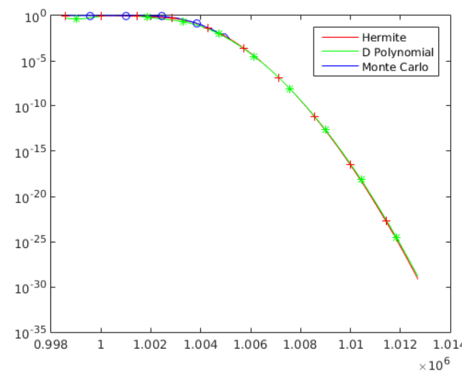

Fig. 2. Complementary CDF comparison from equations (5), (6) for $M = 5$ and $N = 10^6$.

IX. Conclusion

In this paper, new methods for computing the distribution of the largest eigenvalue of a complex Wishart matrix were introduced and demonstrated. This work was motivated by the desire to calculate detection thresholds for problems in passive radar detection, where the GLRT for a rank-one signal in white Gaussian noise is the largest eigenvalue of the Gram matrix constructed from the receiver data. Previously published methods for calculating this distribution will overflow double precision arithmetic for the problem sizes encountered in passive radar due to formulations in terms of ratios of gamma functions. The methods outlined here are derived using relations of the gamma functions to the Laguerre polynomials to eliminate this overflow. Experimental comparisons between the new and old methods as well as Monte Carlo simulations were performed. Future work will focus on generalizing these methods to signals of higher rank, as well as calculating theoretical error bounds.

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