DETECTION IN MULTIPLE CHANNELS HAVING UNEQUAL NOISE POWER

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ABSTRACT

A bayesian detector is formulated for the problem of detecting a signal of known rank using data collected at multiple sensors. The noise on each sensor channel is white and gaussian, but its variance is unknown and may be different from channel to channel. A low-SNR assumption that enables approximation of one of the marginalization integrals in the likelihood ratio, yielding a tractable approximate bayesian detector for this regime. Performance of this detector is evaluated and compared to other recently introduced detectors.

Index Terms— Multiple-channel detection; Bayesian detection; Uncalibrated receivers

1. INTRODUCTION

The problem of detecting the presence of an unknown signal using a suite of several spatially distributed sensors has received considerable attention over the past four decades [1, 2, 3]. Recently, motivated in part by growing interest in passive radar and certain passive electronic surveillance applications, several papers have addressed the situation where the signal rank is known [4, 5, 6, 7, 8] and the "uncalibrated receiver" situation in which the noise power is unknown and not assumed to be equal across the sensor channels [9, 10, 11, 7].

Two basic types of model appear in this body of work: one where the presence of a signal affects the mean of the observed data and the other where the signal manifests in the covariance of the data. With the exception of [6], most of this work develops generalized likelihood ratio tests (GLRTs) or approximations of such tests under specific assumptions such as low signal-to-noise ratio (SNR). In this paper, we adopt a model of the first type for the problem of detecting a signal of known rank when the noise variances on the sensor channels are unknown and not constrained to be equal. This model is set forth precisely in Section 2. We proceed in Section 3 to observe that no GLRT exists for this problem. A bayesian detector is formulated in Section 4, and a low-SNR approximation to this detector is obtained when approximations to some of the marginalization integrals entailed in the likelihood ratio are used. The paper concludes with a brief comparative performance evaluation using simulated data and a discussion of the results.

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2. MODEL AND PROBLEM FORMULATION

We wish to model a scenario in which the presence of an unknown signal of known rank K is to be detected using a suite of $M > K$ spatially distributed sensors. Received data at each sensor are assumed to have been filtered to a band of interest, suitably sampled, and appropriately adjusted in time delay and Doppler to correspond to a physical state of interest for the test. This pre-processing provides M complex vectors, each of length N, which serve as the data for the detector. Precisely, the data are modeled as a $M \times N$ data matrix

$$X = AS + \nu,$$

whose m^{th} row, $\boldsymbol{x}_m$, represents a vector of N samples of the noisy signal collected at the m^{th} sensor. The K-dimensional signal subspace is defined by the matrix $S \in \mathbb{C}^{K\times N}$ whose rows are orthonormal vectors in $\mathbb{C}^N$. The element a_{mk} of the matrix $A \in \mathbb{C}^{M\times K}$ is the complex amplitude of the component of the signal received at sensor m and in the subspace corresponding to the k^{th} row of S. Both A and S are unknown, except for the properties just described. The noise vector ν is assumed to be temporally white in each channel, and the covariance matrix across the channels is $R = \text{diag}(\sigma_1^2, \ldots, \sigma_M^2)$ where σ_m^2 is the unknown receiver noise power associated with the m^{th} sensor.

The detection problem may be formulated as a binary hypothesis test as follows:

$$\begin{aligned} &H_0 : X \sim \mathcal{CN}(0, R) \\ &H_1 : X \sim \mathcal{CN}(AS, R), \text{for some } S \text{ and some non-zero } A. \end{aligned} \tag{1}$$

Under H_0, the joint probability density function (pdf) of X conditioned on R is

$$p(X|H_0, R) = \frac{1}{\pi^{MN}|R|^N} \exp\left(-\text{Tr}(R^{-1}XX^\dagger)\right) \tag{2}$$

where $\dagger$ denotes hermitian transpose and $|R|$ is the determinant of R. Under H_1, the joint pdf of X conditioned on A, S, and R is

$$\begin{aligned} &p(X|H_1, A, S, R) \\ &= \frac{1}{\pi^{MN}|R|^N} \exp\left(-\text{Tr}\left(R^{-1}(X - AS)(X - AS)^\dagger\right)\right). \end{aligned} \tag{3}$$

With this set-up, we proceed to examine the detection problem in the following sections.

3. NONEXISTENCE OF THE GLRT DETECTOR

The generalized likelihood ratio (GLR) is is the ratio of the maximal values over the unknown parameters of the joint likelihood functions under the respective hypotheses H_1 and H_0. From (2) and (3), the GLR in this case is given by

$$\mathcal{L} = \frac{\max_{A,S,\{\sigma_m^2\}} p(X|H_1, A, S, \{\sigma_m^2\})}{\max_{\{\sigma_m^2\}} p(X|H_0, \{\sigma_m^2\})}.$$

For the detection problem (1), the likelihood function under H_1, $p(X|H_1, A, S, \{\sigma_m^2\})$ in the numerator of the GLR, is an unbounded function of the parameters A, S, and $\{\sigma_m^2\}$. As a consequence, the GLR is infinite irrespective of the data.

This can be seen as follows. Choose $S = \hat{S}$ such that one of the channels $\boldsymbol{x}_j$ is in the subspace spanned by the rows of $\hat{S}$. With this choice $\hat{S}^\dagger \hat{S} \boldsymbol{x}_j^\dagger = \boldsymbol{x}_j^\dagger$, since $\hat{S}^\dagger \hat{S}$ is the orthogonal projector onto the subspace spanned by the rows of $\hat{S}$. Further, choosing $A = \hat{A} = X\hat{S}$ gives

$$\begin{aligned} &p(X|H_1, \hat{A}, \hat{S}, \{\sigma_m^2\}) \\ &= \frac{1}{\pi^{MN}} \prod_{m=1}^{M} \sigma_m^{-2N} \exp\left(-\sigma_m^{-2}(\|\boldsymbol{x}_m\|^2 - \boldsymbol{x}_m \hat{S}^\dagger \hat{S} \boldsymbol{x}_m^\dagger)\right) \\ &= \frac{1}{\pi^{MN}} \sigma_j^{-2N} \prod_{\substack{m=1 \\ m \neq j}}^{M} \sigma_m^{-2N} \exp\left(-\sigma_m^{-2}(\|\boldsymbol{x}_m\|^2 - \boldsymbol{x}_m \hat{S}^\dagger \hat{S} \boldsymbol{x}_m^\dagger)\right). \end{aligned}$$

It is now possible to make the likelihood function arbitrarily large by letting $\sigma_j^2 \to 0$. *We conclude that problem* (1) *does not admit a GLRT*.

In [8], Hack et al. proposed a detector for (1) under low-SNR and long-signal assumptions. Their detector is essentially the GLRT for unequal but known noise variances with these variances replaced by estimates

$$\widehat{\sigma_m^2} = \|\boldsymbol{x}_m\|^2/N.$$

The resulting test statistic is

$$h(X) = \sum_{i=1}^{K} \lambda_i \tag{4}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K$ are the K largest eigenvalues of the normalized $M \times M$ Gram matrix $\widehat{G}$ with elements

$$\widehat{G}_{m\ell} = \frac{\boldsymbol{x}_m \boldsymbol{x}_\ell^\dagger}{\|\boldsymbol{x}_m\| \|\boldsymbol{x}_\ell\|}.$$

Empirical results were presented comparing the performance of this detector to that of the generalized coherence (GC) detector [2, 3] in the context of this problem. The GC detection statistic,

$$\gamma_{GC}^2 = |\widehat{G}|$$

is optimal for testing for arbitrary non-diagonal covariance structure across the channels [12]. The simulation results presented also included a detector based on the Frobenius norm of the normalized Gram matrix; i.e.,

$$\gamma_F^2(X) = \|\widehat{G}\|_F^2 = \mathrm{Tr}\left(\widehat{G}^2\right).$$

This detector was derived in [9] for a related problem and recently was shown in [13] to be a locally-most powerful invariant test (LMPIT) for correlation of gaussian vectors.

4. BAYESIAN DETECTOR

In this section, we derive a bayesian detector for unknown and unequal receiver noise power. In this approach, the detector may be written as

$$\mathcal{L} = \frac{\int p(X|H_1, A, S, R) p(A, R) p(S)\, dA\, dS\, dR}{\int p(X|H_0, R) p(R)\, dR} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{5}$$

where $p(A, R)$ and $p(S)$ are the prior pdfs for the nuisance parameters (A, R) and S respectively. The joint prior for R and A is chosen as follows [12, 6]:

$$p(R \,|\, \tau) = \tau^M e^{-\tau \mathrm{Tr}(R^{-1})}, \tag{6}$$

$$p(A \,|\, \beta^2, R) = \frac{1}{\pi^{MK} \beta^{2MK}} |R^{-1}|^K e^{-\frac{1}{\beta^2} \mathrm{Tr}(R^{-1} A A^\dagger)}. \tag{7}$$

Under H_0, marginalizing over R with respect to the prior (6) gives

$$\begin{aligned} &p(X|H_0) \\ &= \frac{\tau^M}{\pi^{MN}} \int |R^{-1}|^N \exp\left(-\mathrm{Tr}(R^{-1}(XX^\dagger + \tau I_M))\right) dR^{-1} \end{aligned} \tag{8}$$

where I_M denotes $M \times M$ identity matrix. The noise variances $\{\sigma_m^2\}_{m=1}^M$ may be expressed as $\sigma_m^2 = \rho_m \sigma^2$ with $\sum_{m=1}^M \rho_m = 1$. With this parameterization, ρ_m is the unknown fraction of the total noise variance σ^2 in the m^{th} channel for for $m = 1, \ldots, M$. It follows that $R = \sigma^2 R_\rho$ where $R_\rho = \mathrm{diag}(\rho_1, \ldots, \rho_M)$; i.e.,

$$p(R^{-1})\, dR^{-1} = \sigma^{-2(M-1)} |R_\rho^{-1}|^2\, d\sigma^{-2}\, dR_\rho. \tag{9}$$

Making this change of variables in (8) gives the posterior pdf under H_0 conditioned on $\boldsymbol{\rho} = (\rho_1, \ldots, \rho_M)$ as

$$\begin{aligned} &p(X|H_0, \boldsymbol{\rho}) \\ &= \frac{\tau^M |R_\rho^{-1}|^{N+2}}{\pi^{MN}} \int \sigma^{-2p-1} \exp\left(-\frac{1}{\sigma^2} \mathrm{Tr}(R_\rho^{-1}(XX^\dagger + \tau I_M)\right) d\sigma \\ &= \frac{\tau^M \Gamma(p) |R_\rho^{-1}|^{N+2}}{\pi^{MN}} \left(\mathrm{Tr}(R_\rho^{-1}(XX^\dagger + \tau I_M))\right)^{-p} \end{aligned} \tag{10}$$

where $p = M(N+1)$.

Under H_1, marginalizing A from (3) with respect to the prior (7) yields

$$\begin{aligned} &p(X|H_1, S, R) \\ &= \frac{|R^{-1}|^{K+N}}{\pi^{MN+MK} \beta^{2MK}} \exp\left(-\mathrm{Tr}(X^\dagger R^{-1} X - \alpha P X^\dagger R^{-1} X)\right) \\ &\times \int_A \exp\left(-\mathrm{Tr}(R^{-1} \alpha^{-1} (A - \alpha X S^\dagger)(A - \alpha X S^\dagger)^\dagger)\right) dA \\ &= \frac{|R^{-1}|^N}{\pi^{MN} (1+\beta^2)^{MK}} \exp\left(-\mathrm{Tr}(X^\dagger R^{-1} X - \alpha P X^\dagger R^{-1} X)\right), \end{aligned}$$

where $\alpha = \beta^2/(1+\beta^2)$ and $P = S^\dagger S$ is an orthogonal projection into the K-dimensional subspace spanned by the rows of S. Similarly, marginalizing R gives

$$\begin{aligned} p(X|H_1, S, \boldsymbol{\rho}) &= \frac{\tau^M |R_\rho^{-1}|^{N+2}}{\pi^{MN} (1+\beta^2)^{MK}} \\ &\times \int \sigma^{-(2p+1)} \exp\left(-\frac{1}{\sigma^2} \mathrm{Tr}(R_\rho^{-1}(XX^\dagger + \tau I_M - \alpha X P X^\dagger))\right) d\sigma \\ &= \frac{p(X|H_0, \boldsymbol{\rho})}{(1+\beta^2)^{MK}} \left(1 - \frac{\alpha \mathrm{Tr}(P W_\rho)}{\mathrm{Tr}(W_\rho) + \tau \mathrm{Tr}(R_\rho^{-1})}\right)^{-p} \end{aligned} \tag{11}$$

where $W_\rho = X^\dagger R_\rho^{-1} X$.

Finally the parameter P may be marginalized by integrating (11) with respect to invariant measure over the grassmannian manifold $\mathbf{G}_{K,N}$, yielding

$$p(X|H_1,\boldsymbol{\rho}) = \frac{p(X|H_0,\boldsymbol{\rho})}{(1+\beta^2)^{MK}} \times \int_{\mathbf{G}_{K,N}} \left(1 - \frac{\alpha \mathrm{Tr}(PW_\rho)}{\mathrm{Tr}(W_\rho) + \tau \mathrm{Tr}(R_\rho^{-1})}\right)^{-p} d\mu(P) \tag{12}$$

As described in [6], this integral can be well approximated using Laplace approximation:

$$\int_{\mathbf{G}_{K,N}} \left(1 - \frac{\alpha \mathrm{Tr}(PW_\rho)}{\mathrm{Tr}(W_\rho) + \tau \mathrm{Tr}(R_\rho^{-1})}\right)^{-p} d\mu(P) \approx \frac{(\pi/p\alpha)^q}{\mathrm{vol}(G_{K,N})} (1 - \alpha \sum_{i=1}^{K} \tilde{\lambda}_i(\boldsymbol{\rho}))^{q-p} \prod_{i=1}^{K} \prod_{j=1}^{N-K} \frac{1}{\tilde{\lambda}_i(\boldsymbol{\rho}) - \tilde{\lambda}_j(\boldsymbol{\rho})}.$$

In this expression,

$$\tilde{\lambda}_i(\boldsymbol{\rho}) = \frac{\lambda_i(\boldsymbol{\rho})}{\sum_{i=1}^{N} \lambda_i(\boldsymbol{\rho}) + \tau \mathrm{Tr}(R_\rho^{-1})} \tag{13}$$

with $\lambda_1(\boldsymbol{\rho}) > \lambda_2(\boldsymbol{\rho}) > \cdots > \lambda_N(\boldsymbol{\rho})$ the eigenvalues of W_ρ, $q = K(N-K)$, and $\mathrm{vol}(G_{K,N})$ the volume of the grassmannian $G_{K,N}$; i.e.,

$$\mathrm{vol}(G_{K,N}) = \frac{(2\pi)^{K(N-K)} \prod_{\ell=1}^{K-1} \ell!}{\prod_{\ell=N-K}^{N-1} \ell!}.$$

The posterior pdf under H_1 is thus

$$p(X|H_1,\boldsymbol{\rho}) \approx p(X|H_0,\boldsymbol{\rho}) \frac{(\pi/p\alpha)^q}{(1+\beta^2)^{MK} \mathrm{vol}(G_{K,N})} \times (1 - \alpha \sum_{i=1}^{K} \tilde{\lambda}_i)^{q-p} \prod_{i=1}^{K} \prod_{j=1}^{N-K} \frac{1}{\tilde{\lambda}_i(\boldsymbol{\rho}) - \tilde{\lambda}_j(\boldsymbol{\rho})}. \tag{14}$$

The nuisance parameter remaining to be marginalized is $\boldsymbol{\rho}$. This is achieved by integrating over $\boldsymbol{\rho}$; i.e., the bayesian likelihood ratio is

$$\mathcal{L} = \frac{\int p(X|H_1,\boldsymbol{\rho})\, d\boldsymbol{\rho}}{\int p(X|H_0,\boldsymbol{\rho})\, d\boldsymbol{\rho}} \tag{15}$$

where both integrals are over the $(M-1)$-simplex.

4.1. Approximate Bayesian Detector

Exact computation of the integrals in (15) has not yet been carried out. In view of this, we propose the following approximation for the low-SNR regime. With the change of variable from (9),

$$\begin{aligned} p(\sigma_1^2,\ldots,\sigma_M^2) &= p(\sigma^2,\boldsymbol{\rho}) d\sigma^{-2} d\boldsymbol{\rho} \\ &= p(\sigma^2|\boldsymbol{\rho}) p(\boldsymbol{\rho}) d\sigma^{-2} d\boldsymbol{\rho}. \end{aligned} \tag{16}$$

For low SNR, we make the crude approximation $p(\boldsymbol{\rho}) = \delta(\boldsymbol{\rho} - \hat{\boldsymbol{\rho}})$ where

$$\hat{\rho}_m = \|\boldsymbol{x}_m\|^2 / \mathrm{Tr}(XX^\dagger).$$

Using this approximation prior (16), the likelihood ratio

$$\mathcal{L}_{\hat{\rho}} = \frac{p(X|H_1,\hat{\boldsymbol{\rho}})}{p(X|H_0,\hat{\boldsymbol{\rho}})}$$

where $P(X|H_0,\hat{\boldsymbol{\rho}})$ and $P(X|H_1,\hat{\boldsymbol{\rho}})$ are as given (10) and (14) with $\hat{\boldsymbol{\rho}}$ in place of ρ. Explicitly, we have

$$\mathcal{L}_{\hat{\rho}} \approx \frac{(\pi/p\alpha)^q (1 - \alpha \sum_{i=1}^{K} \tilde{\lambda}_i(\hat{\boldsymbol{\rho}}))^{q-p}}{(1+\beta^2)^{MK} \mathrm{vol}(G_{K,N})} \prod_{i=1}^{K} \prod_{j=1}^{N-K} \frac{1}{\tilde{\lambda}_i(\hat{\boldsymbol{\rho}}) - \tilde{\lambda}_j(\hat{\boldsymbol{\rho}})} \tag{17}$$

where $\tilde{\lambda}_i(\hat{\boldsymbol{\rho}})$ is as defined in (13) and $\lambda_i(\hat{\boldsymbol{\rho}}) > \cdots > \lambda_N(\hat{\boldsymbol{\rho}})$ are eigenvalues of $X^\dagger R_{\hat{\rho}}^{-1} X$, with $R_{\hat{\rho}} = \mathrm{diag}(\hat{\rho}_1,\ldots,\hat{\rho}_M)$. In the performance results presented in Section 5, we refer to the detector based on the statistic as the "approximate bayesian detector."

5. PERFORMANCE

This section compares the detection performance of the proposed bayesian detector (17) with the detector (4) of Hack et al. [8] and the Frobenius norm detector described in [9, 13] for the detection problem (1). The performance of the GLRT with *known* but unequal noise power on the channels is also shown as a baseline. The test statistic for this detector is $\sum_{i=1}^{K} \lambda_i$, where $\lambda_1,\ldots,\lambda_K$ are the K largest eigenvalues of $XX^\dagger$.

As in [8], the simulation was carried out with $M = 6$ receivers and signal length $N = 1000$ samples. For each realizaton, the transmitted signals are randomly generated as $S = \exp(i\theta)$ where the random variables θ are i.i.d. and uniformly distributed on $[0, 2\pi]$. The coefficient A is drawn from a Gaussian distribution and scaled to achieve a desired average input SNR, defined by $\mathrm{SNR}_{\mathrm{avg}} = \frac{1}{M}\sum_{m=1}^{M} \mathrm{SNR}_m$. Also as in [8], we compare the performance of these detectors with fixed probability of false alarm $P_f = 10^{-3}$ with the variance of the noise on the m^{th} receiver drawn independently from a uniform distribution; i.e., $\sigma_m^2 \sim U(-3,3)$dB. For each $\mathrm{SNR}_{\mathrm{avg}}$, probabilities of detection are averaged over $50,000$ realizations. For the bayesian detector, the parameters are $\beta^2 = 10^6$ and $\tau = 10$.

Figure 1 shows curves of probability of detection (P_d) as a function of average input SNR for all four detectors listed above. Curves for signal ranks of $K = 1$ (top), $K = 2$ (middle), and $K = 4$ (bottom) are presented. Figure 2 shows ROC curves for all four detectors operating in the same scenario with input average SNR$= -12$dB for ranks $K = 1$ and $K = 2$ and average SNR$= -10$dB for rank $K = 4$.

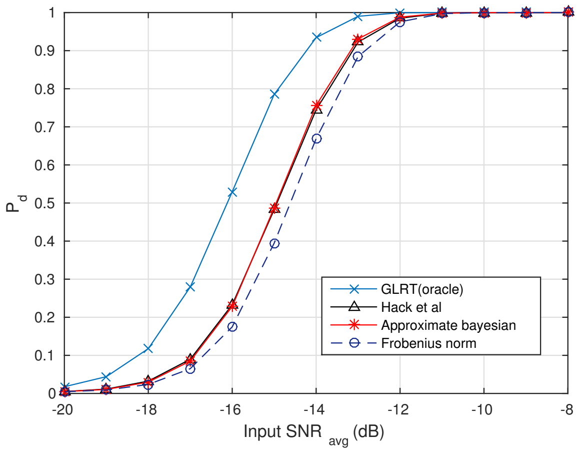
6. DISCUSSION AND CONCLUSION

This paper has established the framework of a bayesian test for multichannel detection of an unknown rank-K signal in the so-called uncalibrated receiver situation; i.e., where the noise power on the channels is unknown and not necessarily equal across the channels. The realization here entails two approximations:

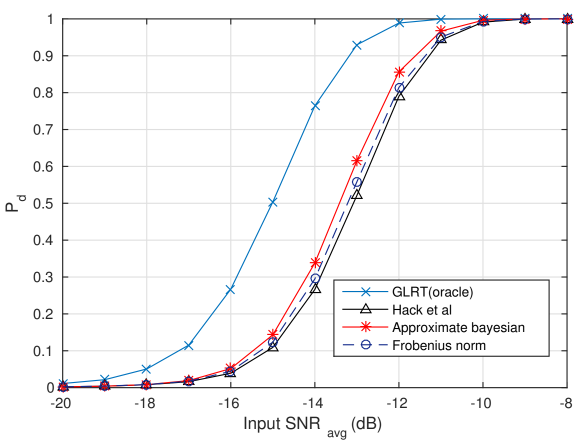
1. Laplace approximation of the integral (12).
2. The low-SNR approximation described in Section 4.1.

Nevertheless, the detector obtained outperformed the detector (4) in [8] in all tests undertaken on scenario (1). The margin be is most evident for higher signal ranks. The detector introduced here also consistently outperformed the Frobenius norm detector, with the margin (unsurprisingly) larger for low signal ranks.

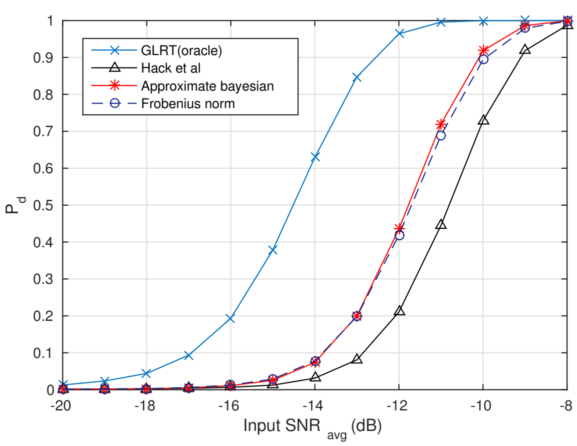
Ongoing work of the authors has produced an exact value for (12), and we are seeking exact solutions to the integrals in (15) as well. We anticipate that replacing these approximations with exact solutions will provide improved detection performance, particularly at higher SNRs.



(a) signal rank $K = 1$

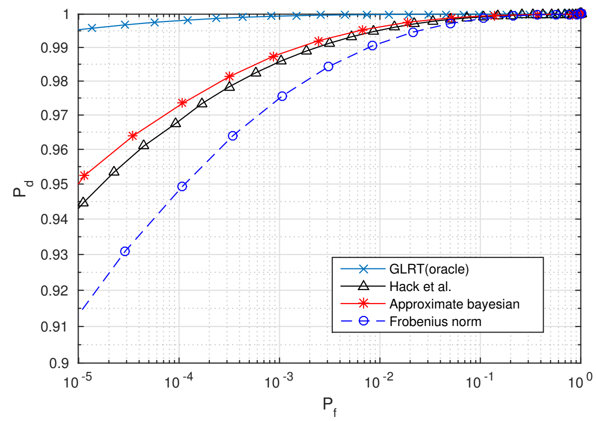


(b) signal rank $K = 2$

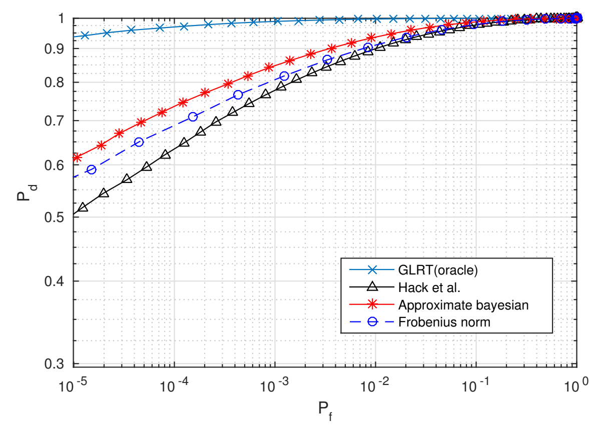


(c) signal rank $K = 4$

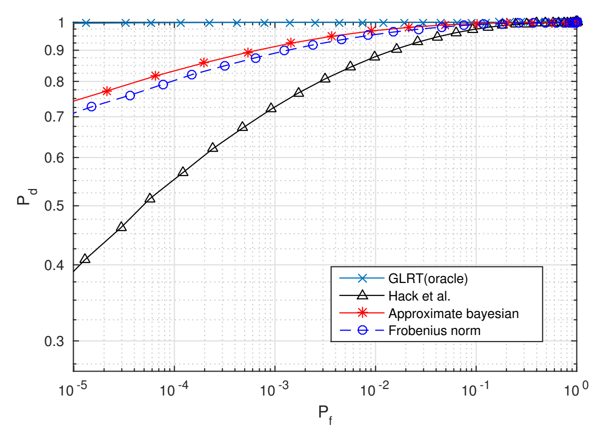
Fig. 1: Probability of detection as a function of average SNR for four detectors. The test scenario uses $M = 6$ channels with data vector length $N = 1000$. Plots for signal ranks of $K = 1$ (top), $K = 2$ (middle), and $K = 4$ (bottom) are shown.



(a) signal rank $K = 1$, input $\text{SNR}_{\text{avg}} = -12\text{dB}$



(b) signal rank $K = 2$, input $\text{SNR}_{\text{avg}} = -12\text{dB}$



(c) signal rank $K = 4$, input $\text{SNR}_{\text{avg}} = -10\text{dB}$

Fig. 2: ROC curves for four detectors. The test scenario uses $M = 6$ channels with data vector length $N = 1000$. Plots are shown for signal rank of $K = 1$ with average SNR of -12 dB (top), rank $K = 2$ with average SNR of -12 dB (middle), and rank $K = 4$ with average SNR of -10dB (bottom).

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