Detection and Characterization of MIMO Radar Signals

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***Abstract*—Motivated by electronic surveillance applications, this paper considers the problems of detecting the presence of and characterizing a radar transmitter using data collected at a spatially distributed suite of receivers. A characterization of particular interest is determining the rank of the transmitted signal, which enables discrimination between MIMO and conventional radar transmitters as well as distinguishing between MIMO systems that simultaneously emit different numbers of linearly independent signals from their transmit arrays. Bayesian detectors are derived and their performance is demonstrated in simulations. Generalized likelihood ratio tests are also derived and some drawbacks they manifest in this setting are noted.**

I. Introduction

Numerous multiple-input, multiple-output (MIMO) radar concepts have been explored in research literature over the past decade [1]–[4]. Among these is the idea of using a transmit array consisting of $K \geq 2$ closely-spaced elements that emit linearly independent waveforms.[1] In what follows, Friedlander's convention of reserving the term *MIMO radar* for schemes involving spatial waveform diversity in a phased array radar, while using *multistatic radar* to mean that transmit and receive components are spatially distributed [6].

This paper takes the perspective of electronic surveillance (ES), where typical goals are to detect and characterize radio frequency (RF) signals and to localize their sources using passive means. Specifically, it is assumed that data are collected by a suite of M spatially distributed receivers with the objectives of detecting and characterizing radar transmissions. Although the elements in the transmit array may be well separated in terms of the wavelength of the carrier frequency, in the far-field they are essentially co-located on the scale of wavelengths present in the baseband waveform. So, as discussed in more detail in Sec. III, K linearly independent transmit waveforms will define a K-dimensional subspace of the space of signals collected across the receiver array.

The problem of detecting a common but unknown signal of known rank K using data collected at M spatially distributed receivers has been addressed in recent signal processing literature [7], [8]. This paper extends the approaches developed in [8] to the problem of MIMO radar signal characterization. Specifically, statistical tests are developed for discriminating between a signal of known rank K and a noise-only null hypothesis. These tests lead to tests for a MIMO (i.e., rank $K \geq 2$) signal versus a conventional (i.e., rank-one) signal, for rank-K versus rank-J MIMO signals, and for a signal rank estimator, all using data collected at $M > K$ spatially distributed receivers.

Notation: In this paper, vectors are denoted by bold lowercase symbols and bold uppercase symbols are used for matrices. T denotes transpose, $\dagger$ Hermitian transpose, Tr trace, and $\mathbf{I}_n$ the $n \times n$ identity matrix.

II. ES View of MIMO Radar

Consider a scenario in which a MIMO radar with a K-element transmit array simultaneously emits K linearly independent waveforms, $\boldsymbol{s}(t) = (s_1(t), s_2(t), \ldots, s_K(t))^{\mathsf{T}}$, where each $s_k(t)$ represents a complex baseband signal. Let $\boldsymbol{a}(\boldsymbol{k}) = (a_1(\boldsymbol{k}), a_2(\boldsymbol{k}), \ldots, a_K(\boldsymbol{k}))^{\mathsf{T}}$ denote the transmit array steering vector in the direction $\boldsymbol{k}$. In the far field, the transmitted waveform in the direction $\boldsymbol{k}$ can be modeled as

$$y(t) = \boldsymbol{a}(\boldsymbol{k})^{\dagger}\boldsymbol{s}(t)e^{2\pi i f_c t}$$

where f_c is the carrier frequency of the radar. The signal transmitted by this MIMO radar impinges on $M > K$ spatially distributed receivers of a surveillance system. The baseband signal collected at the m^{th} receiver is

$$x_m(t) = \alpha_m \boldsymbol{a}(\boldsymbol{k}_m)^{\dagger}\boldsymbol{s}(t-\tau_m)e^{2\pi i f_m t} + \nu_m(t),$$

where $\alpha_m \in \mathbb{C}$ accounts for path loss and antenna gain for the m^{th} receiver and includes the carrier phase. In this expression, $\tau_m = ||\boldsymbol{r}_m - \boldsymbol{r}_0||/c$ is the travel time from the transmit array at position $\boldsymbol{r}_0$ to the m^{th} receiver at position $\boldsymbol{r}_m$ and c denotes the speed of light. The Doppler shift at the m^{th} receiver is $f_m = -f_c(\boldsymbol{v}_m - \boldsymbol{v}_0) \cdot \boldsymbol{k}_m/c$, where $\boldsymbol{v}_0$ is the velocity of the transmitter and $\boldsymbol{v}_m$ the velocity of the receiver. Finally, $\nu_m(t)$ is additive white Gaussian noise on receiver channel m which is assumed to be uncorrelated from channel to channel.

III. Problem Formulation

Suppose there is a MIMO radar transmitter with $K > 1$ elements at a position $\boldsymbol{r}_0$ and with velocity $\boldsymbol{v}_0$. Then, for $m =$

[1] Some highly credible radar experts have pointed out that the practical advantages of such MIMO radar approaches have probably been overstated in much of the research literature on the subject, e.g., [5]. While the authors agree with this assessment, the intent here is to examine the detectability of MIMO radar signals from an electronic surveillance perspective rather than to remark on other merits of MIMO radar.

$1,\dots,M$,

$$\boldsymbol{x}_m(t-\tau_1+\tau_m)=\alpha_m\boldsymbol{a}(\boldsymbol{k}_m)^\dagger\boldsymbol{s}(t-\tau_1)e^{2\pi i f_m t}+\nu_m(t-\tau_1+\tau_m) \quad (1)$$

This expression shows that, when suitably adjusted to account for time differences of arrival and Doppler shifts, the M received signal components span a K-dimensional space defined by a common time-frequency shift of the K transmitted signals. The problem of detecting the presence of an emitter at a given candidate location $\boldsymbol{r}_0$ and velocity $\boldsymbol{v}_0$ is addressed in [8] and further examined below. Two additional objectives of interest for ES in this scenario are: (1) test whether the emitter is a MIMO (rank $K>1$) transmitter versus a conventional (rank $K=1$) transmitter; and (2) estimate the number K of linearly independent waveforms being transmitted.

The tests developed here thus concern the hypotheses

H_K : A rank-K transmitter is at position $\boldsymbol{r}_0$ with velocity $\boldsymbol{v}_0$,

for $K=1,2,\dots$. The following paragraph describes how the hypothesis H_K manifests in the test data.

Each received signal $x_m(t)$ is shifted to baseband, compensated for time delays and Doppler shifts corresponding to the values of $\boldsymbol{r}_0$ and $\boldsymbol{v}_0$ being considered in the test, and sampled at an appropriate rate to obtain a set of M complex N-vectors. These vectors of received data are grouped into an $M\times N$ matrix $\mathbf{X}$. Define $\mathbf{S}$ to be a $K\times N$ matrix whose rows are orthonormal (i.e., $\mathbf{S}\mathbf{S}^\dagger=\mathbf{I}_K$) and span the signal subspace. Then under H_K, $\mathbf{X}$ can be written as

$$\mathbf{X}=\mathbf{A}\mathbf{S}+\mathbf{N} \quad (2)$$

where $\mathbf{A}$ is a complex $M\times K$ matrix with elements $\mathbf{A}_{ij}=\alpha_i a_j(\boldsymbol{k}_i)$ and the elements of the noise matrix $\mathbf{N}$ are independent with $\mathbf{N}_{ij}\sim\mathcal{CN}(0,\sigma^2)$. The $K\times N$ matrix $\mathbf{S}$ defines the K-dimensional signal subspace. The values of $\mathbf{A},\mathbf{S}$ and σ^2 are assumed to be unknown.

The first problem considered in this paper is testing the hypothesis H_K that a phased array MIMO radar is at $(\boldsymbol{r}_0,\boldsymbol{v}_0)$ against the alternative hypothesis that the collected signal is just receiver noise; i.e., the detection problem is

$$\begin{aligned} H_K &: \mathbf{X}=\mathbf{A}_K\mathbf{S}_K+\mathbf{N} \\ H_0 &: \mathbf{X}=\mathbf{N} \end{aligned} \quad (3)$$

The likelihood ratio for testing H_K versus H_J may be obtained as the quotient of the likelihood ratios for the tests of H_K versus H_0 and H_J versus H_0. Thus solving (3) also addresses the problem of distinguishing H_K from H_J. This allows estimation of K in an obvious way, which is discussed explicitly in Sec. IV-C and illustrated in Sec. V.

IV. RANK-K MIMO RADAR DETECTION

In this section the likelihood ratio for the detection problem (3) is derived using a Bayesian approach for elimination of the nuisance parameters. The generalized likelihood ratio test (GLRT) for this problem is also discussed briefly. Under H_0, the probability density function (pdf) of $\mathbf{X}$ conditioned on σ^2 is

$$p(\mathbf{X}|\sigma^2)=(\pi\sigma^2)^{-MN}e^{-\frac{N}{\sigma^2}\mathrm{Tr}(\mathbf{W})} \quad (4)$$

where

$$\mathbf{W}=\frac{1}{N}\mathbf{X}^\dagger\mathbf{X}.$$

Under H_K the pdf of $\mathbf{X}$ conditioned on $\mathbf{A}_K,\mathbf{S}_K$ and σ^2 is

$$\begin{aligned} &p(\mathbf{X}|\mathbf{A}_K,\mathbf{S}_K,\sigma^2) \\ &=(\pi\sigma^2)^{-MN}e^{-\frac{1}{\sigma^2}\mathrm{Tr}(\mathbf{X}-\mathbf{A}_K\mathbf{S}_K)(\mathbf{X}-\mathbf{A}_K\mathbf{S}_K)^\dagger} \\ &=(\pi\sigma^2)^{-MN}e^{-\frac{N}{\sigma^2}\mathrm{Tr}((\mathbf{I}_N-\mathbf{P}_V)\mathbf{W})}e^{-\frac{1}{\sigma^2}\mathrm{Tr}((\mathbf{A}_K-\mathbf{X}\mathbf{S}_K^\dagger)(\mathbf{A}_K-\mathbf{X}\mathbf{S}_K^\dagger)^\dagger} \end{aligned} \quad (5)$$

where $\mathbf{P}_V=\mathbf{S}_K^\dagger\mathbf{S}_K$ is the orthogonal projection onto the subspace V spanned by the rows of $\mathbf{S}_K$.

A. Generalized likelihood ratio test

The GLRT is obtained by considering the ratio of maximal values of the joint likelihood functions with respect to the unknown or nuisance parameters under hypotheses H_0 and H_K as follows:

$$\frac{\max_{\mathbf{A}_K,\mathbf{S}_K,\sigma^2}p(\mathbf{X}|\mathbf{A}_K,\mathbf{S}_K,\sigma^2)}{\max_{\sigma^2}p(\mathbf{X}|\sigma^2)}\gtrless_{H_0}^{H_K}\gamma. \quad (6)$$

Maximizing the likelihood functions (4) and (5) with respect to $\mathbf{A}_K,\sigma^2$ and $\mathbf{S}_K$ and substituting the maximum values back into (6), yields (see [8] for the details of the maximization with respect to $\mathbf{P}_V$)

$$\mathrm{GLR}=\left(1-\frac{\sum_{i=1}^K\lambda_i}{\sum_{i=1}^N\lambda_i}\right)^{-MN}$$

where $\lambda_1>\lambda_2>\cdots>\lambda_N$ are eigenvalues of $\mathbf{W}$. Note that the non-zero eigenvalues of $\mathbf{W}$ are exactly the eigenvalues of the sample-covariance matrix $\hat{\mathbf{R}}=\mathbf{X}\mathbf{X}^\dagger/N$. The value of $\mathbf{P}_V$ which maximizes the numerator of (6) is the maximum likelihood estimator of $\mathbf{P}_V$,

$$\hat{\mathbf{P}}_V=\sum_{k=1}^K\boldsymbol{v}_k\boldsymbol{v}_k^\dagger \quad (7)$$

where $\{\boldsymbol{v}_k|k=1,\cdots,K\}$ are the normalized eigenvectors of $\mathbf{W}$ corresponding to its K largest eigenvalues. An unfortunate property of this generalized likelihood ratio is that as the hypothesized rank K is increased, the hypotheses H_K become increasingly likely relative to H_0. It is shown below that the Bayesian detector is much better behaved in this respect.

B. Bayesian Test and Rank Posterior

In the Bayesian approach, instead of formulating maximum-likelihood estimates of the nuisance parameters $\mathbf{P}_V$ and σ^2, each nuisance parameter is marginalized out of the likelihood function by integration with respect to an appropriate prior probability distribution. That is,

$$\frac{p(\mathbf{X}|H_K)}{p(\mathbf{X}|H_0)}=\frac{\int p(\mathbf{X}|\mathbf{A}_K,\mathbf{P}_V,\sigma^2)p(\mathbf{A}_K)p(\sigma^2)d\mathbf{A}_K d\sigma^{-2}d\mu(\mathbf{P}_V)}{\int p(\mathbf{X}|\sigma^2)p(\sigma^2)d\sigma^{-2}}$$

where the integrals are over the appropriate parameter spaces. First, using the uniform prior for A_K, i.e., $p(\mathbf{A}_K)=1$,

$$p(\mathbf{X}|\mathbf{P}_V,\sigma^2)=(\pi\sigma^2)^{-M(N-K)}e^{-\frac{N}{\sigma^2}\mathrm{Tr}((\mathbf{I}_N-\mathbf{P}_V)\mathbf{W})} \quad (8)$$

Using the prior pdf for σ^2 as given in [8],

$$p(\sigma^{-2}\mathbf{I}_M) = \tau^{Mq}\Gamma^{-M}(q)\sigma^{-2M(q-1)}e^{-\tau M\sigma^{-2}}$$

then under H_0, the marginalized (with respect to σ^{-2}) likelihood is

$$p(\mathbf{X}|H_0,q,\tau) = \frac{\tau^{Mq}\Gamma(p)}{\pi^{MN}\Gamma^M(q)}\left(N(\mathrm{Tr}(D)+\frac{M\tau}{N})\right)^{-p}$$

where $p = M(N+q-1)+1$, $q > 1-N-1/M$ and D is the diagonal matrix of eigenvalues $\lambda_1 > \lambda_2 > \cdots > \lambda_N$ of $\mathbf{W}$.

The grassmannian space $G_{K,N}$ is the space of all K-dimensional subspace of $\mathbb{C}^N$. It is a smooth complex manifold of complex dimension $K(N-K)$. Each orthogonal projector $\mathbf{P}_V$ can be identified with a point in $G_{K,N}$. The non-informative prior distribution for $\mathbf{P}_V$ can be taken as the invariant measure on grassmannian $G_{K,N}$. Under H_K the likelihood marginalized with respect to σ^{-2} and $\mathbf{P}_V$ is

$$p(\mathbf{X}|H_K) = \frac{\tau^{Mq}\Gamma(p-MK)}{\pi^{M(N-K)}\Gamma^M(q)}\times \int_{G_{K,N}}\left(N(\mathrm{Tr}(D)-\mathrm{Tr}(D\mathbf{P}_V)+\frac{M\tau}{N})\right)^{-(p-MK)} d\mu(\mathbf{P}_V)$$

where $d\mu(\mathbf{P}_V)$ denotes the invariant measure on $G_{K,N}$. The likelihood ratio becomes, upon setting $q = 1$ and taking the limit $\tau \to 0$,

$$\frac{p(\mathbf{X}|H_K)}{p(\mathbf{X}|H_0)} = c(\mathrm{Tr}(D))^{MK}\int_{G_{K,N}}(1-\mathrm{Tr}(\tilde{D}\mathbf{P}_V))^{-M(N-K)-1}d\mu(\mathbf{P}_V) \tag{9}$$

where $c = (N\pi)^{MK}\Gamma(M(N-K)+1)/\Gamma(MN+1)$ and

$$\tilde{D} = D/\mathrm{Tr}(D).$$

To compute the integral, it is necessary to parameterize $\mathbf{P}_V$ in terms of local coordinates on $G_{K,N}$. Using the results in [9], [8]

$$\mathbf{P}_V = \begin{pmatrix}(\mathbf{I}_K+\mathbf{Z}^\dagger\mathbf{Z})^{-1} & (\mathbf{I}_K+\mathbf{Z}^\dagger\mathbf{Z})^{-1}\mathbf{Z}^\dagger \\ \mathbf{Z}(\mathbf{I}_K+\mathbf{Z}^\dagger\mathbf{Z})^{-1} & \mathbf{Z}(\mathbf{I}_K+\mathbf{Z}^\dagger\mathbf{Z})^{-1}\mathbf{Z}^\dagger\end{pmatrix} \tag{10}$$

where $\mathbf{Z} \in \mathbb{C}^{(N-K)\times K}$. In these coordinates the normalized invariant measure on $G_{K,N}$ takes the form

$$d\mu(\mathbf{P}_V) = \frac{1}{\mathrm{vol}(G_{K,N})}\det(\mathbf{I}_K - \mathbf{Z}^\dagger\mathbf{Z})^{-N}d\mathbf{Z}$$

where

$$d\mathbf{Z} = \prod_{i=1}^{N-K}\prod_{j=1}^{K} d\mathrm{Re}\,(z_{ij})d\mathrm{Im}\,(z_{ij})$$

and $\mathrm{vol}(G_{K,N})$ denotes the volume of the grassmannian

$$\mathrm{vol}(G_{K,N}) = \frac{\prod_{\ell=N-K+1}^{N} A_{2\ell-1}}{\prod_{\ell=1}^{K} A_{2\ell-1}} \tag{11}$$

where A_ℓ is the area of the unit sphere in $\mathbb{R}^\ell$,

$$A_\ell = \frac{2\pi^{\ell/2}}{\Gamma(\ell/2)}.$$

Substituting (10)-(11) into (9) gives

$$\frac{c(\mathrm{Tr}(D))^{MK}}{\mathrm{vol}(G_{K,N})}\int_{\mathbf{Z}\in\mathbb{C}^{(N-K)\times K}} e^{-p'\log(1-\mathrm{Tr}(\tilde{D}\mathbf{P}_V))-N\log\det(\mathbf{I}_K+\mathbf{Z}^\dagger\mathbf{Z})}d\mathbf{Z}$$

where $p' = M(N-K)+1$. The integral can be approximated, using Laplace approximation along with some matrix identities which are given in [8], as

$$\frac{c(\mathrm{Tr}(D))^{MK}}{\mathrm{vol}(G_{K,N})(1-\mathrm{Tr}(\tilde{D}_K))^{p'}}\prod_{i=1}^{N-K}\prod_{j=1}^{K}\int_{z_{ij}} e^{-\gamma p'(\tilde{\lambda}_i-\tilde{\lambda}_{K+j}+\delta)|z_{ij}|^2}dz_{ij}$$

where $\delta = \frac{N}{\gamma p}$, $\gamma = (1-\mathrm{Tr}(\tilde{D}_K))^{-1}$ and $\tilde{D}_K = D_K/\mathrm{Tr}(D)$ where D_K is diagonal matrix with the first K eigenvalues of D and $\tilde{\lambda} = \lambda/\sum_{i=}^{N}\lambda_i$. The likelihood ratio is then

$$\frac{p(\mathbf{X}|H_K)}{p(\mathbf{X}|H_0)} = c'\left(\sum_{i=1}^{N}\lambda_i\right)^{MK}\left(1-\sum_{i=1}^{K}\tilde{\lambda}_i\right)^{-p'+K(N-K)} \times \prod_{i=1}^{N-K}\prod_{j=1}^{K}\frac{1}{\tilde{\lambda}_i-\tilde{\lambda}_{K+j}+\delta} \tag{12}$$

where $c' = c(\frac{\pi}{p'})^{K(N-K)}/\mathrm{vol}(G_{K,N})$.

C. Posterior distribution for the rank of the radar

The likelihood ratios given in equation (12) can be used to construct the posterior distribution for K. Denoting the prior probabilities for H_K, $K = 0,\cdots,M-1$ as $p(H_K)$, the probability of a rank-K transmitter at position $\boldsymbol{r}_0$ is given by

$$p(K|\boldsymbol{r}_0) = \begin{cases}\frac{p(H_0)}{p(H_0)+\sum_{K=1}^{M-1}p(K)p(\mathbf{X}|K,\boldsymbol{r}_0)/p(\mathbf{X}|H_0)}, & \text{if } K=0\\ \frac{p(H_K)p(\mathbf{X}|H_K,\boldsymbol{r}_0)/p(\mathbf{X}|H_0)}{p(H_0)+\sum_{K=1}^{M-1}p(H_K)p(\mathbf{X}|H_K,\boldsymbol{r}_0)/p(\mathbf{X}|H_0)}, & \text{otherwise.}\end{cases} \tag{13}$$

Note that, with M receivers, only signals with rank up to $K = M-1$ can be distinguished.

V. SIMULATIONS

In this section, the performance of the Bayesian rank-K signal detector derived above is evaluated through simulation. A rank-4 MIMO radar simultaneously transmitting four orthogonal waveforms is simulated. The transmitter consists of a four-element uniform linear array with half wavelength spacing at 9.4GHz carrier frequency. The waveform consists of a CPI of sixteen, four-channel binary phase coded pulses with a pulse repetition interval (PRI) of $100\mu s$. Each pulse has a code consisting of a Hadamard sequence (row of a Hadamard matrix) of length 128 with chip length of $100ns$. The four channels have mutually orthogonal Hadamard sequences. The ES receiver system consists of eight sensors distributed as shown in Figure 1. The total distance spanned by the receivers is 5.454 kilometers and with the most distant receiver at range 4.862km from the radar. The power of the received signals is set relative to the shortest ranges from the radar to the closest sensor, which is 3.910 kilometers and is normalized to unit

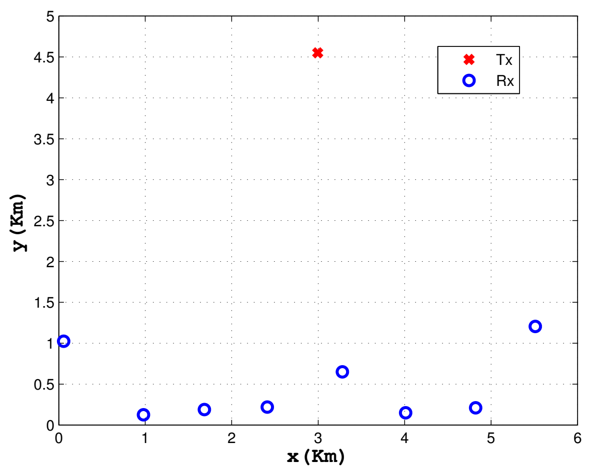

Fig. 1. Configuration of transmitter and receivers for the simulation.

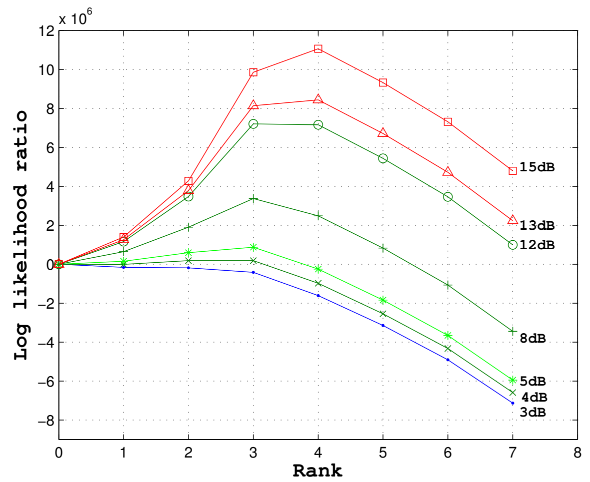

Fig. 2. Log-likelihood ratio values for the hypothesized signal rank $K = 1, \dots, 7$ at different SNRs.

power. The SNR is defined by $\text{SNR} = -10 \log_{10} \sigma^2$, where σ^2 is noise power per complex dimension. For simplicity, the transmitters and receivers are assumed to be mutually stationary.

Figure 3 shows plots of the likelihood ratio (12) at 14dB SNR for rank $K = 1, 2, \dots, 7$. The results show that the detector is able to detect signals at the transmitter's position at $(2.995, 4.550)$ and correctly identify the signal rank as $K = 4$. The last sub-figure shows plots of log probability ratio (13) which clearly demonstrate that the highest probability occurs at rank 4.

Figure 4 shows the performance of the detector in the same scenario as in Fig. 3, except that the signal rank is now 1; i.e., it is a conventional phased array radar with all four elements transmitting the same waveform. The simulation was carried out with phased coded waveform with the same properties as in the rank-4 signal simulation, but at 5dB SNR. As shown, the detector is able to identify/distinguish between rank-1 and rank-$K, K > 1$. The last sub-figure shows the plots of log probability ratio (13) which clearly demonstrates the highest probability occurs at rank-1.

Figure 2 shows results obtained from the same scenarios as in Fig. 3 for a range of SNRs from 3 to 15dB. The figure shows the plots of the expected log likelihood ratio, averaged over 500 realizations in each case, against rank at the different SNRs. For this scenario, the detector is able to identify as rank-4 signal if SNR $\geq$ 13dB.

VI. Conclusion

An important message to take from the analysis of the MIMO radar detector developed in this paper is that, if one wants to accurately determine the presence and rank of such a radar, one needs numerous ES receivers or high SNR as the rank increases. For a MIMO radar of rank K, at least $K + 2$ receivers are needed. Further, as K increases, the receivers need to have wide spatial separation in order to ascertain the rank of the radar. Of course, in ES applications, one would also like to estimate the subspace spanned by the signals transmitted by the radar. The current analysis suggests methods for achieving this (Eq. (10)), and this topic will be taken up in a subsequent publication. The problem of determining the actual signals transmitted by the radar rather than just the subspace they span appears daunting, as this requires simultaneous determination of the configuration and orientation of the radar's transmit array.

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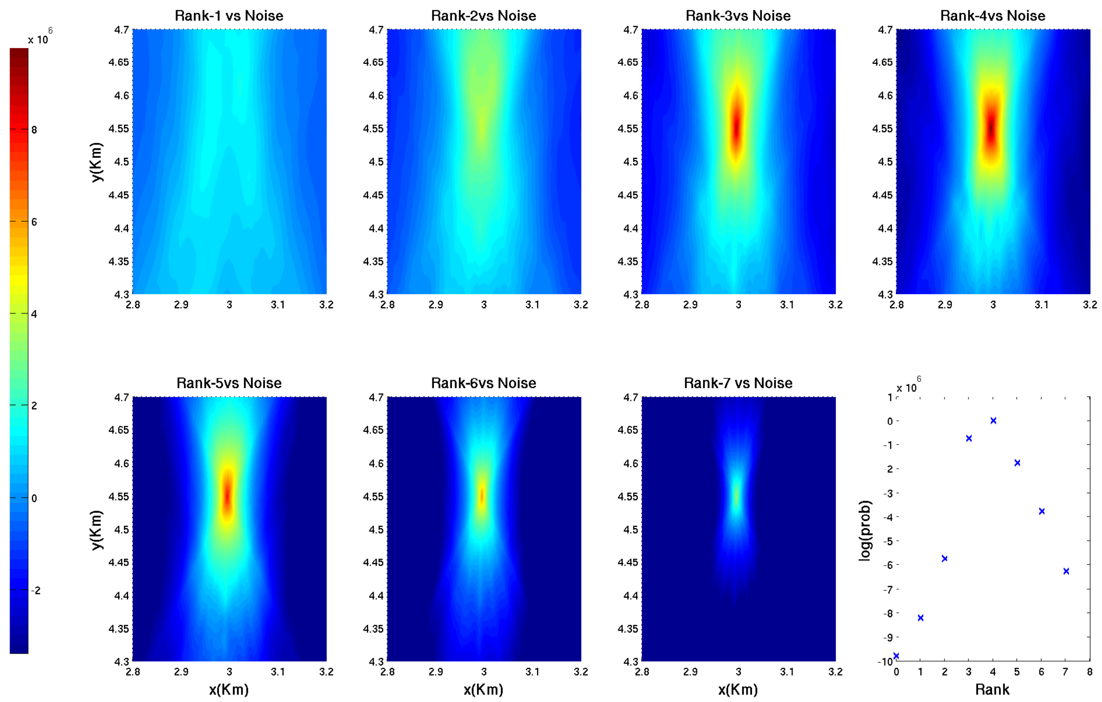

Fig. 3. Plots of likelihood ratio between the rank-K signal hypothesis H_K, $K = 1, \ldots, 7$, and the noise-only hypothesis H_0 for a rank-4 (MIMO) phased array radar signal.

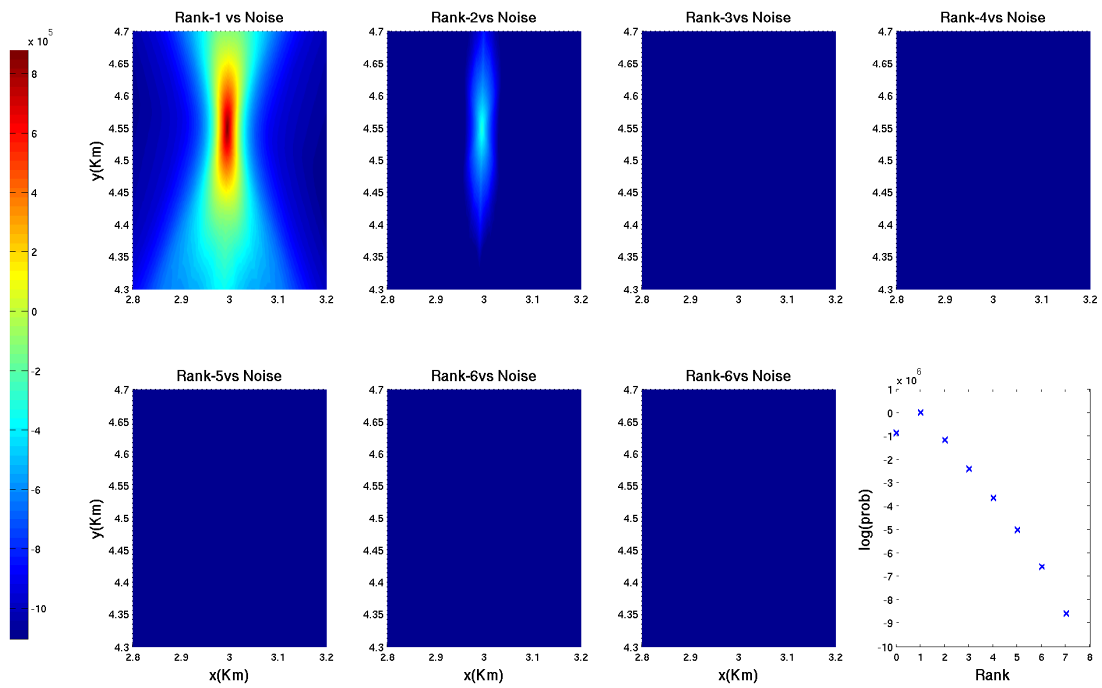

Fig. 4. Plots of likelihood ratio between the rank-K signal hypothesis H_K, $K = 1, \ldots, 7$, and the noise-only hypothesis H_0 for a rank-1 (conventional) phased array radar signal.