Detection in Networked Radar

Kaitlyn Beaudet[1] Lauren Crider[2] and Douglas Cochran[1,2]

[1]School of Electrical, Computer & Energy Engineering
Arizona State University
Tempe AZ 85287-5706 USA

[2]School of Mathematical & Statistical Sciences
Arizona State University
Tempe AZ 85287-1804 USA

ABSTRACT

The potential applicability of multiple-channel coherence estimation in situations where one channel contains a noise-free signal replica (as in active radar) or a high-SNR reference signal (as in passive coherent radar) has been proposed in recent work. Invariance of the distribution of M-channel coherence estimate statistics, including recently derived variants optimized for detection of signals having known rank, to the presence of a strong signal on one channel provided all channels are independent and the other $M-1$ channels contain only noise enables the desired use of these statistics without altering detection thresholds designed to provide desired false-alarm probabilities. Traditionally, multiple-channel detection using coherence estimates has assumed that time series data from all channels are aggregated at a fusion center. Mitigation of this requirement to demand global aggregation of only scalar statistics that can be computed locally by sharing of data between pairs of nodes has been explored, and the use of maximum-entropy methods to provide surrogate statistics for pairs of nodes that are not in direct communication within a network has been proposed for traditional passive detection problems. This paper examines the applicability of this idea in the presence of a strong reference channel with particular attention to ascertaining relationships between network topology and detection performance.

Keywords: Multistatic radar, MIMO radar, Passive Coherent Radar, Multi-channel detection

1. INTRODUCTION

Recent and continuing advances in networking technologies have precipitated new interest in multistatic and so-called multiple-input, multiple-output (MIMO) radar architectures. This paper is concerned with situations in which data collected in a network of $M \geq 2$ spatially distributed sensors is to be exploited in the detection and localization of objects in a scene. In each processing epoch, the scene is illuminated by a single transmitter that emits a known waveform. The sensors receive direct path signal energy as well as energy scattered from objects in the scene, and time-shifted and Doppler-shifted replicates of the transmitted signal are to be detected in the collected data. A similar situation occurs in passive coherent radar, the scene is illuminated by a transmitter of opportunity and the direct-path signal plays the role of the transmitted signal replicate in processing.[1] This signal is often strong and may also be possible to de-noise by decoding its message content and then re-creating a close replica of the transmitted signal for use in extracting returns from the scatterers, which are generally much weaker.

With this motivation, Section 2 begins by considering the problem of detecting a known signal in several channels of noisy data under the assumption that all the data are available, together with the known signal, in one place (i.e., a "fusion center") for processing. A generalized likelihood ratio test (GLRT) is derived and compared to a multiple-channel detector based on generalized coherence (GC) estimation.[2,3] Section 3 examines detection performance in incomplete networks where surrogate statistics for channels that are not in direct communication are provided by maximum-entropy methods. Detection performance for a detector based on GC estimation is compared to a detector based on the GLRT for various SNR values on the channels that are not in direct connect with the transmitter. These results are discussed in Section 4.

Kaitlyn Beaudet, Kaitlyn.Beaudet@asu.edu; Lauren Crider, lcrider@asu.edu; Douglas Cochran, cochran@asu.edu

Algorithms for Synthetic Aperture Radar Imagery XX, edited by Edmund Zelnio,
Frederick D. Garber, Proc. of SPIE Vol. 8746, 87460V · © 2013 SPIE
CCC code: 0277-786X/13/$18 · doi: 10.1117/12.2020670

2. DETECTION WITH FULLY AGGREGATED DATA

The detection problem described in Section 1 is formulated mathematically as follows. Suppose a suitable sampling rate and integration interval have been chosen and the sampled transmitted signal is represented as a deterministic vector $\mathbf{s} \in \mathbb{C}^N$. It will be assumed, as is normally true in practice, that $N \gg M$, where M is the number of sensors. The sampled sensor data, which have each been appropriately compensated for relative time delay and Doppler shift to account for a putative scatterer position and motion, are also represented by complex N-vectors $\mathbf{x}_1, \ldots, \mathbf{x}_M$. The null hypothesis is defined as

$$H_0 \;:\; \mathbf{x}_k = \mathbf{w}_k, \quad k = 1, \ldots, M \tag{1}$$

where the random vectors $\mathbf{w}_k$ are independent complex, zero-mean, white Gaussian noise; i.e., $\mathbf{w}_k \sim \mathcal{CN}[\mathbf{0}, \sigma^2\mathbb{I}]$ with $\mathbf{0}$ the zero vector in $\mathbb{C}^N$ and $\mathbb{I}$ the $N \times N$ identity matrix. The alternative hypothesis is

$$H_1 \;:\; \mathbf{x}_k = \alpha_k \mathbf{s} + \mathbf{w}_k, \quad k = 1, \ldots, M$$

where each α_k is an unknown complex gain.

When the transmitted signal $\mathbf{s}$ and all the data $\mathbf{x}_k$, $k = 1, \ldots, M$ are available in a common location for processing, several standard tests are available for discriminating between H_0 and H_1. The following subsection develops the generalized likelihood ratio test for this situation. Subsequently, a test based on the generalized coherence estimate is described. GC-based detectors are most often associated with passive detection; e.g., ascertaining the presence of a common but unknown signal in multiple noisy channels of sensor data. The idea of using GC-based detection in active settings by treating the transmitted signal as one channel (with infinite SNR) was broached in 1997[4] and recently developed more fully.[5] This section of the paper closes with a comparison of detection performance between these two approaches based on simulations and a discussion of the results.

2.1 Multi-channel GLRT

With all data available in a single location for processing, it is possible to aggregate them into a single complex NM-vector $\mathbf{X}$ defined by

$$\mathbf{X}^\mathsf{T} = [\mathbf{x}_1^\mathsf{T} \;\cdots\; \mathbf{x}_M^\mathsf{T}]^\mathsf{T}$$

Under H_0,

$$\mathbf{X} = \begin{bmatrix} \mathbf{w}_1 \\ \vdots \\ \mathbf{w}_M \end{bmatrix}$$

and its probability density function is

$$p(\mathbf{X}|H_0) = \frac{1}{\pi^{NM}\sigma^{2NM}} \exp\left\{-\frac{1}{\sigma^2}\mathbf{X}^\dagger\mathbf{X}\right\} = \frac{1}{\pi^{NM}\sigma^{2NM}} \exp\left\{-\frac{1}{\sigma^2}\sum_{k=1}^{M}\mathbf{x}_k^\dagger\mathbf{x}_k\right\}$$

where $\dagger$ denotes hermitian transpose. Under H_1,

$$\mathbf{X} = \begin{bmatrix} \alpha_1\mathbf{s} + \mathbf{w}_1 \\ \vdots \\ \alpha_M\mathbf{s} + \mathbf{w}_M \end{bmatrix}$$

and its probability density function is

$$p(\mathbf{X}|H_1) = \frac{1}{\pi^{NM}\sigma^{2NM}} \exp\left\{-\frac{1}{\sigma^2}\sum_{k=1}^{M}(\mathbf{x}_k - \alpha_k\mathbf{s})^\dagger(\mathbf{x}_k - \alpha_k\mathbf{s})\right\} \tag{2}$$

Hence the log-likelihood ratio is

$$\ell(\mathbf{X}) = \log\frac{p(\mathbf{X}|H_1)}{p(\mathbf{X}|H_0)} = \frac{1}{\sigma^2}\sum_k(\overline{\alpha}_k\mathbf{s}^\dagger\mathbf{x}_k + \alpha_k\mathbf{x}_k^\dagger\mathbf{s} - |\alpha_k|^2||\mathbf{s}||^2) \tag{3}$$

where $\overline{\alpha}_k$ denotes the complex conjugate of α_k and $||\mathbf{s}||$ is the norm of the vector $\mathbf{s}$.

The parameter σ^2 (the noise variance) is assumed known, so the signal-to-noise ratios (SNRs) on the channels are controlled by the unknown gain parameters $\alpha_1, \ldots, \alpha_M$. To calculate the generalized likelihood ratio (GLR), it is necessary to estimate these unknown parameters. From (2), the log-likelihood function for the α_k has the form

$$C - \frac{1}{\sigma^2} \sum_k (\mathbf{x}_k^\dagger \mathbf{x}_k - \overline{\alpha}_k \mathbf{s}^\dagger \mathbf{x}_k - \alpha_k \mathbf{x}_k^\dagger \mathbf{s} + |\alpha_k|^2 ||\mathbf{s}||^2),$$

which is maximized by minimizing

$$\sum_k [\mathbf{x}_k^\dagger \mathbf{x}_k - \overline{\alpha}_k \mathbf{s}^\dagger \mathbf{x}_k - \alpha_k \mathbf{x}_k^\dagger \mathbf{s} + |\alpha_k|^2 ||\mathbf{s}||^2] = ||\mathbf{s}||^2 \sum_k \left[\left(\overline{\alpha_k - \frac{\mathbf{s}^\dagger \mathbf{x}_k}{||\mathbf{s}||^2}} \right) \left(\alpha_k - \frac{\mathbf{s}^\dagger \mathbf{x}_k}{||\mathbf{s}||^2} \right) - \mathbf{x}_k^\dagger \mathbf{x}_k - \frac{s^\dagger \mathbf{x}_k \mathbf{x}_k^\dagger \mathbf{s}_k}{||\mathbf{s}||^2} \right]$$

The maximum-likelihood (ML) estimate of α_k is thus

$$\hat{\alpha}_k = \frac{\mathbf{s}^\dagger \mathbf{x}_k}{||\mathbf{s}||^2}$$

Substituting these ML estimates for the into the log-likelihood ratio (3) gives the log GLR as

$$\frac{1}{\sigma^2} \sum_{k=1}^{M} \left(\mathbf{s}^\dagger \mathbf{x}_k \mathbf{x}_k^\dagger \mathbf{s} \right) = \frac{1}{\sigma^2} \sum_k |\mathbf{s}^\dagger \mathbf{x}_k|^2$$

so the GLRT may be written as $\rightarrow H_1 = \{\sum |\mathbf{s}^\dagger \mathbf{x}_k|^2 > T\}$ where T is a threshold.

2.2 Active detection with generalized coherence

The GC estimate for $M \geq 2$ data channels $\mathbf{x}_1, \ldots, \mathbf{x}_M$ is defined as[2,3]

$$\hat{\gamma}^2(\mathbf{x}_1, \ldots, \mathbf{x}_M) = 1 - \frac{\det G(\mathbf{x}_1, \ldots, \mathbf{x}_M)}{||\mathbf{x}_1||^2 \cdots ||\mathbf{x}_M||^2}$$

where

$$G(\mathbf{x}_1, \ldots, \mathbf{x}_M) = \begin{bmatrix} \langle \mathbf{x}_1, \mathbf{x}_1 \rangle & \langle \mathbf{x}_1, \mathbf{x}_2 \rangle & \cdots & \langle \mathbf{x}_1, \mathbf{x}_M \rangle \\ \langle \mathbf{x}_2, \mathbf{x}_1 \rangle & \langle \mathbf{x}_2, \mathbf{x}_2 \rangle & \cdots & \langle \mathbf{x}_2, \mathbf{x}_M \rangle \\ \vdots & & \ddots & \vdots \\ \langle \mathbf{x}_M, \mathbf{x}_1 \rangle & \langle \mathbf{x}_M, \mathbf{x}_2 \rangle & \cdots & \langle \mathbf{x}_M, \mathbf{x}_M \rangle \end{bmatrix}$$

is the $M \times M$ Gram matrix whose entries are the inner products $\langle \mathbf{x}_i, \mathbf{x}_j \rangle = \mathbf{x}_j^\dagger \mathbf{x}_i$. Although comparison of the GC estimate to a threshold actually discriminates between a diagonal covariance matrix and an arbitrary non-diagonal covariance matrix,[6] this test is often used to ascertain the presence of a common but unknown signal on several noisy channels.[3] The distribution of $\hat{\gamma}^2$ under H_0 is known,[2,3] allowing detection thresholds corresponding to desired false alarm probabilities to be determined analytically. Invariance results for the distribution of the Gram matrix determinant under H_0,[4,7,8] and indeed for any function of the eigenvalues of the Gram matrix determinant under H_0,[5] show that these thresholds remain valid if one of the M channels has arbitrary distribution provided the other $M-1$ channels are distributed according to the H_0 model (1) and all M channels are independent. Thus, detection thresholds determined for the H_0 model (1) can still be used if one channel consists of a noiseless replica of the transmitted signal as long as the other channels contain only zero-mean white Gaussian noise and all channels are independent.

Based on this observation, an alternative to the GLRT derived in Section 2.1 is

$$\rightarrow H_1 = \{\hat{\gamma}^2(\mathbf{s}, \mathbf{x}_1, \ldots, \mathbf{x}_M) > T\}$$

This detector was initially proposed in 1997[4] and was recently shown to give the same performance as a matched filter detector having the same invariance properties in the case $M = 1$.[5]

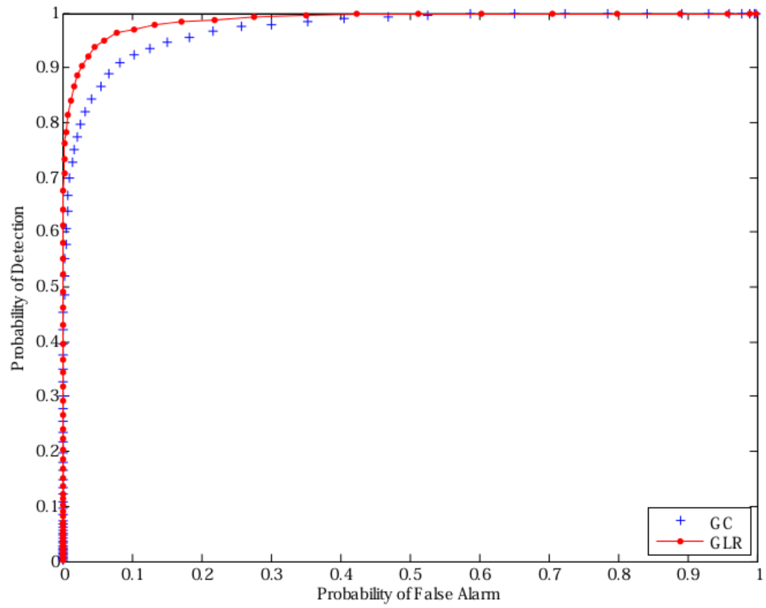

Figure 1: ROC curves for GLRT (red dots) and GC (blue crosses) detectors using fully aggregated data from three receivers. Receivers A, B, and C all have SNR of -27dB.

2.3 Detector comparison with fully aggregated data

Although the GLRT has no optimality guarantees, it is generally accepted to be a sound test for this application. The GC-based method is not designed for this application, and it is not expected to perform as well as the GLRT when all sensor data are available at the fusion center. In Section 3, however, the GC approach is shown to offer implementation advantages in distributed radar networks that are not completely connected.

Figure 1 shows receiver operating characteristic (ROC) curves for the GLRT and GC-based detectors using fully aggregated data from $M = 3$ receivers. The signal is a complex sinusoid, the complex data vectors are of length $N = 2048$, and all three receiver channels have SNR of -27 dB. As anticipated, the GLRT provides superior performance to that of the MC detector in this situation.

3. DETECTION IN AN INCOMPLETELY CONNECTED NETWORK

Computing either the GLRT or GC estimate $\hat{\gamma}^2(\mathbf{s}, \mathbf{x}_1, \ldots, \mathbf{x}_M)$ depends on the availability of the inner products $\langle \mathbf{x}_k, \mathbf{s} \rangle$, $k = 1, \ldots, M$, between the transmitted signal $\mathbf{s}$ and the receiver data from each of the M receivers. The GC estimate also incorporates inner products $\langle \mathbf{x}_i, \mathbf{x}_j \rangle$, $i \neq j$, between the data at every pair of distinct receivers. Consider the network depicted in Figure 2(a). In this diagram, the circle represents the transmitter and each square a receiver. Each pair of elements connected by an edge in this network graph are assumed to be in direct communication, and consequently able to form the inner product of their data locally. Only these scalar values are assumed to be communicated to the fusion center. Compared to an architecture that accumulates all the raw data at the fusion center, this scheme substantially reduces the communication bandwidth between the network and the fusion center by performing data reduction locally between pairs nodes that are directly connected to each other in the network. So, in the network of Figure 2(a), both the GLRT and the MSC detector can be implemented at the fusion center. For the network configuration in Figure 2(b), the transmitted signal is shared only with Receiver C. With this configuration, the fusion center does not receive all the data necessary to implement either test. The largest GLRT it can perform only uses sensor data from Receiver C. The GC estimate cannot be computed because the inner products $\langle \mathbf{s}, \mathbf{x}_A \rangle$ and $\langle \mathbf{s}, \mathbf{x}_B \rangle$ are missing.

The following subsection describes an approach for completing the Gram matrix and performing an approximate GC test when the network is incompletely connected. Simulation results are presented that compare the performance of the largest GLRT enabled by the network topology with this approximate GC test.

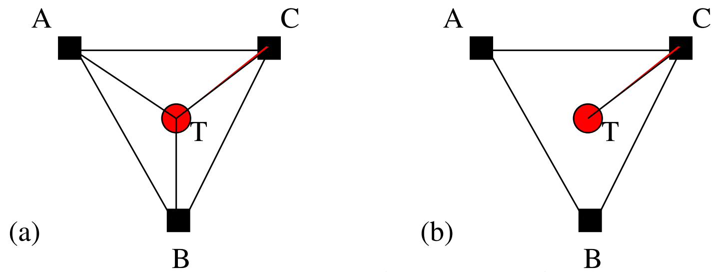

Figure 2: Two networks, each with three receiver nodes (black squares) and one transmitter node (red circle). (a) The network is completely connected; i.e., every node shares data directly with every other node. (b) An incompletely connected network in which all three receivers share data directly, but only Receiver C shares data directly with the transmitter and hence has access to the transmitted sequence.

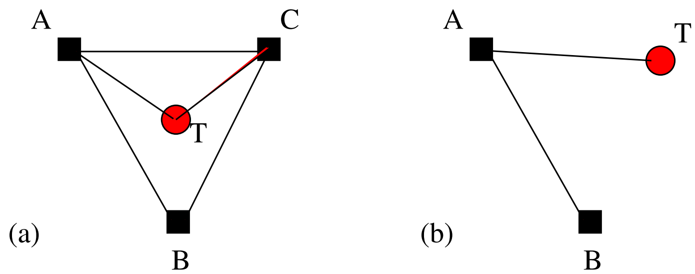

Figure 3: Two additional incompletely connected networks used for detector performance evaluation. (a) A network with three receivers (black squares) and one transmitter node (red circle). All three receivers are share direct communication links, but only receivers A and C communicate directly with the transmitter. (b) A network with two receivers and one transmitter. Receiver A communicates directly with the transmitter and with Receiver B, but Receiver B does not communicate directly with the transmitter.

3.1 Maximum-entropy surrogation in GC-based detection

When the network is not completely connected, as in the case depicted in Figure 2(b), the fusion center cannot form the Gram matrix $G(\mathbf{s}, \mathbf{x}_1, \dots, \mathbf{x}_M)$ because the inner product values corresponding to missing edges in the network graph are not available. A method for forming an approximate GC estimate in this situation was recently introduced in the context of passive detection.[9] This approach uses the principle of maximum entropy[10] to introduce surrogate values for the missing inner products.

Briefly, the method assumes that there is a complex random variable x_k associated with each network node, modeling data samples collected at that node. Collected samples at node k are hence realizations of x_k and can be used to estimated the mean and variance of x_k in standard ways. The ability to communicate between nodes i and j linked by an edge permits estimation of the covariance $\text{cov}(x_i, x_j)$. For a complete network, it is thus possible to estimate the full $M \times M$ covariance matrix C of the variables $x_1, \dots, x_M$. Assuming the x_k all have mean zero, if N independent samples of x_k are collected at node k for each k and are assimilated into complex N-vectors $\mathbf{x}_1, \dots, \mathbf{x}_M$ of sample values, then the standard estimator $\widehat{C}$ of C is proportional to the Gram matrix $G(\mathbf{x}_1, \dots, \mathbf{x}_M)$ used in the GC estimate. The GC detector can thus be viewed as a test on the estimated covariance matrix of the variates $x_1, \dots, x_M$. If each $\mathbf{x}_k$ is further assumed to be normalized to unit variance, using the true variance in place of an estimate on the main diagonal gives a test matrix of the form

$$G\left(\frac{\mathbf{x}_1}{||\mathbf{x}_1||}, \dots, \frac{\mathbf{x}_M}{||\mathbf{x}_M||}\right)$$

The maximum-entropy method holds that missing values in C should be surrogated in such a way as to introduce no new assumptions about the nature of the random variables or of the network. The joint distribution of the random variables $x_1, \dots, x_M$ that best describes current knowledge (i.e., the covariance estimates for all pairs of directly connected nodes) with no further assumptions is the maximum entropy distribution constrained by the available data. The problem of finding the maximum-entropy completion of a covariance matrix has been studied in prior literature, where it is noted that the covariance matrix of this maximum-entropy distribution will have the property that its inverse will has zeros in positions corresponding to the missing covariance values.[12] This observation gives a direct means for calculating the needed surrogate values in small networks. The necessary surrogate values for the small networks considered in this paper may be calculated directly in this manner. Finding k surrogate values in this way becomes prohibitively cumbersome for $M > 5$. Fortunately, such maximum-entropy covariance matrix completion problems fall into a class of determinant maximization problems (entropy in this setting is $\log\det \hat{C}$) that can be efficiently solved by convex programming techniques.[11,12]

3.2 Detection performance in incompletely connected networks

Figures 4 and 5 show ROC curves for the same sinusoid in Gaussian noise problem described in Section 2.3. Figure 4 corresponds to Figure 2(b), an incompletely connected network in which only Receiver C shares data directly with the transmitter. The GC estimate is calculated using surrogated values obtained with the maximum entropy method to for the inner products between the transmitted signal and the Receiver A data and between the transmitted signal and Receiver B data. Since only Receiver C has access to the transmitted signal, the GLRT is calculated using only data from Receiver C. The presence of the common signal $\mathbf{s}$ in noise on Receivers A and B allows the GC detector to outperform the GLRT, even though neither of these receivers has direct access to $\mathbf{s}$ from the transmitter. The bottom curve indicates that even quite weak signals on Receivers A and B are sufficient to endow the GC detector with similar performance to the GLRT in this example.

Figure 5 corresponds to Figure 3(a), again an incompletely connected network in which Receiver B does not communicate directly with the transmitter. In this configuration, Receiver A and Receiver B both have access to $\mathbf{s}$ from the transmitter. So the GLRT is calculated using data from both Receiver A and Receiver B. The GC estimate is calculated using a surrogated inner product value between $\mathbf{s}$ and the Receiver B data. In this case, the performance of the GLRT and the GC detectors is approximately equal when all three receivers have the same SNR. But a stronger signal on Receiver B elevates the performance of the GC detector above that of the GLRT despite Receiver B's lack of direct access to $\mathbf{s}$ from the transmitter.

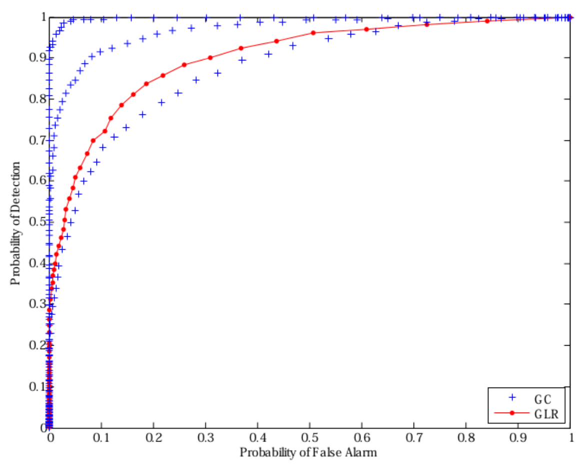

Figure 4: ROC curves for detection of a 2048-component complex sinusoidal signal vector in white complex Gaussian noise in a 4-node sensor network corresponding to Figure 2(b). The red line with dots represents the GLRT and the curves denoted by the blue crosses represent the GC detector with surrogated values for the links between the transmitter and Receiver A and between the transmitter and Receiver B. In all cases, Receiver C has an SNR of -27 dB. The top, middle, and bottom GC curves represent SNRs of - 24 dB, -27 dB, and -33 dB, respectively, on both Receiver A and Receiver B.

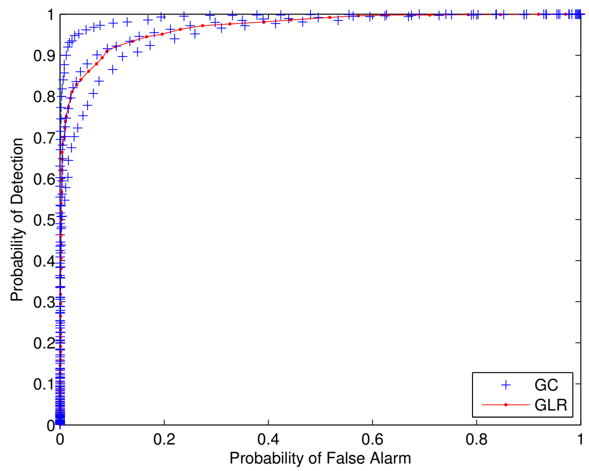

Figure 5: ROC curves for detection of a 2048-component complex sinusoidal signal vector in white complex Gaussian noise in a 4-node sensor network corresponding to Figure 3(a). The red line with dots represents the GLRT and the curves denoted by blue crosses represent the GC detector with surrogated values for the links between the transmitter and Receiver B. In all cases, both Receiver A and Receiver C have SNRs of -27 dB. The top, middle, and bottom GC curves represent SNRs of -24 dB, -27 dB, and -30 dB, respectively, on Receiver B.

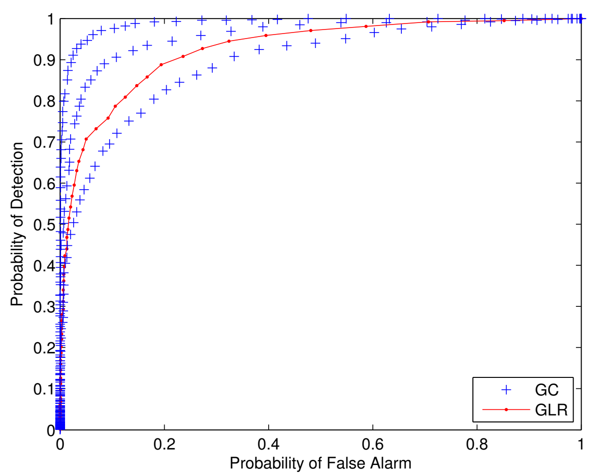

Figure 6: ROC curves for detection of a 2048-component complex sinusoidal signal vector in white complex Gaussian noise in a three-node sensor network. Corresponding to Figure 3(b), the red curve with dots is from the GLRT and the blue curve with crosses is from the GC detector computed using a surrogated value for a link between the transmitter and receiver B. In all cases, Receiver A has an SNR of -27 dB. The top, middle, and bottom GC curves represent SNRs of -24 dB, -27 dB, and -33 dB, respectively, on Receiver B.

Figure 6 shows an ROC curve for detection of a complex sinusoid in white complex Gaussian noise in a three-node sensor network with one transmitter and two receivers. The vectors are of length $N = 2048$. The GC estimate is calculated using surrogated value between the transmitter and Receiver B. Since Receiver B does not have access to the transmitted sequence, as it does not share direct communication, the GLRT is calculated using only data from Receiver A. .

4. DISCUSSION AND CONCLUSIONS

Motivated by the objective of localizing high-bandwidth communication requirements in a radar network, this paper has investigated a multiple-receiver detection scheme that does not require aggregation of raw data at a fusion center. In this scheme, only scalar statistics computed locally from raw data shared between pairs of nodes that are in direct communication in the network are transmitted to the fusion center. For incomplete networks, a maximum-entropy method can be used to provide surrogate statistics for pairs of nodes that are not in direct communication with each another.

Of particular interest in this investigation was the question of whether the presence of noisy replicas of the transmitted signal on some receivers could be of value in detection, even when these receivers lack direct access to the transmitted signal. Four different network topologies were considered and, in each case, a detector based on the GC estimate was compared to a GLRT using data only from receivers having direct access to the transmitted signal. The detector based on the GC estimate uses maximum entropy estimation to provide surrogate values for nodes (transmitter or receiver) that are not in direct connection with each other.

When the network is completely connected, the GLRT unsurprisingly provides better performance. The performance comparison was repeated for two incompletely connected three-receiver networks and one incompletely connected two-receiver network. In all of these cases, the GC-based detector was able to outperform the corresponding GLRT by exploiting noisy information about the transmitted signal obtained from receivers lacking direct access to the transmitted signal.

These experimental findings are quite limited, and only very small networks were examined. The results show sufficient promise that a more comprehensive study, including both theoretical and empirical performance evaluation and involving larger networks, appears warranted.

ACKNOWLEDGMENTS

This work has been supported in part by the University of Michigan and the U.S. Army Research Office under MURI award No. W911NF-11-1-0391, by the U.S. Air Force Office of Scientific Research under Grant No. FA9550-12-1-0225, and by the Fulton Undergraduate Research Initiative (FURI) program at Arizona State University. The authors are grateful to Songsri Sirianunpiboon and Stephen Howard of the Australian Defence Science and Technology Organisation and to Louis Scharf of Colorado State University for their ongoing collaboration in this vein of research.

REFERENCES

[1] Palmer, J., Palumbo, S., Summers, A., Merretta, D., Searle, S., and Howard, S., "An overview of an illuminator of opportunity passive radar research project and its signal processing research directions," *Digital Signal Processing* **21**, 593–599 (September 2011).

[2] Gish, H. and Cochran, D., "Generalized coherence," in [*Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*], 2745–2748 (April 1988).

[3] Cochran, D., Gish, H., and Sinno, D., "A geometric approach to multiple-channel signal detection," *IEEE Transactions on Signal Processing* **43**(9), 2049–2057 (1995).

[4] Clausen, A. and Cochran, D., "An invariance property of the generalized coherence estimate," *IEEE Transactions on Signal Processing* **45**(4), 1065–1067 (1997).

[5] Beaudet, K. and Cochran, D., "Multiple-channel detection in active sensing," in [*Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*], (May 2013).

[6] Sirianunpiboon, S., Howard, S. D., and Cochran, D., "A Bayesian derivation of generalized coherence detectors," in [*Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*], 3253–3256 (March 2012).

[7] Nuttall, A. H., "Invariance of distribution of coherence estimate to second-channel statistics," *IEEE Transactions on Acoustics, Speech, and Signal Processing* **29**(2), 120–122 (1981).

[8] Gish, H. and Cochran, D., "Invariance of the magnitude-squared coherence estimate with respect to second-channel statistics," *IEEE Transactions on Acoustics, Speech, and Signal Processing* **35**(12), 1774–1776 (1987).

[9] Cochran, D., Howard, S. D., Moran, B., and Schmitt, H. A., "Maximum-entropy surrogation in network signal detection," *Proceedings of the IEEE Statistical Signal Processing Workshop* (August 2012).

[10] Jaynes, E. T., "Where do we stand on maximum entropy?," in [*The Maximum Entropy Formalism*], Levine, R. D. and Tribus, M., eds., 15–104, MIT Press (1979).

[11] Boyd, S. and Vandenberghe, L., [*Convex Optimization*], Cambridge University Press (2004).

[12] Vandenberghe, L., Boyd, S., and Wu, S.-P., "Determinant maximization with linear matrix inequality constraints," *SIAM Journal on Matrix Analysis and Applications* **19**(2), 499–533 (1998).