Multi-Mode Detection with Markov Target Motion

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Abstract - ***This paper addresses the problem of configuration of a detection system offering multiple modes of operation that differ in their detection performance and geographical coverage. A technique for optimal mode selection based upon minimizing Bayesian risk is formulated and demonstrated for the case of a two-mode system with a moving target. The dynamics of the target are described by a Markov model.***

Keywords: Attentive sensing, sensor management, detection, resource allocation.

1 Introduction

A recent trend in systems for detection, localization, and classification of targets is the availability of multiple sensing modalities that differ in such crucial measures as detection/discrimination performance, geographical coverage, cost/risk of operation, and time required to gather necessary data. Exploiting the availability of multi-mode and configurable sensor systems in optimal ways for target detection and classification is the central theme of this work.

This paper considers a situation in which a detection system offers multiple modes of operation that differ in their detection performance and geographical coverage. The development that follows focuses on the case of a detector with two operating modes: a "broad search" mode that provides wide coverage and a "focused" mode that provides better detection performance but covers less area. The system is invoked in a sequence of tests to detect and localize a moving target within a framework that is formulated precisely in the following section.

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2 Mathematical Formulation

The situation described above is modeled as follows. The entire region of interest $\mathcal{C}$ is partitioned into N disjoint cells $\mathcal{C}_1, ..., \mathcal{C}_N$. Operating in the broad search mode (*Mode A*), the detector tests for the presence of a signal source in $\mathcal{C}$. In the focused mode (*Mode B*), however, the test may be applied to exactly one cell $\mathcal{C}_n$.

To account for difference in detector performance in the two operating modes, detector performance is modeled as arising from the problem of detecting a known signal in white Gaussian noise of known variance. This model provides a well understood solution (i.e., the matched filter) in each test, admits several straightforward generalizations, and allows detection performance in *Mode B* to be distinguished from that in *Mode A* by simply raising the signal-to-noise ratio (SNR). More specifically, in each mode of operation the detector encounters a problem of the form

$$\begin{aligned} H_0 &: \mathbf{X} = \mathbf{N} \\ H_1 &: \mathbf{X} = S + \mathbf{N} \end{aligned} \tag{1}$$

where S is a known signal M-vector with energy $||S||^2 = 1$ and $\mathbf{N}$ is a zero-mean white Gaussian M-vector having known variance σ^2; i.e., $\mathbf{N} \sim \mathcal{N}[0, \sigma^2 \mathbf{I}]$ where $\mathbf{I}$ is the $M \times M$ identity matrix. Since $||S||$ is fixed, the SNR (and hence the performance of the detector) in each mode can be adjusted by varying σ^2.

Assuming at most one signal source is present, denote by H_1 and H_0 the events that the signal source is, respectively, present in and absent from $\mathcal{C}$. Let $h_0 = H_0$ and, for $n = 1, ..., N$, denote by h_n the event that the signal source is present in cell $\mathcal{C}_n$. With these definitions, $H_1 = \cup_{n=1}^{N} h_n$. Regardless of whether it is operating in *Mode A* or *Mode B*, the system yields a decision $\rightarrow h_n$ with $0 \leq n \leq N$.

Recall that the optimal solution, in terms of minimal probability of error, to a detector problem of the form (1) is a test on the inner product $S^T\mathbf{X}$

where the detection threshold is a function of the *a priori* probability that a signal is present [2, 5]. The probabilities of detection and false alarm for each test are given by error functions of the detection thresholds. In particular, the tests applied in both operating modes will be of this form, but their detection thresholds and probabilities of detection and false alarm will all be different (even when *Mode B* is applied to different cells) because of their dependence on $\Pr(h_n)$, $n = 0, ..., N$.

3 A Bayesian Risk Formulation

Using the notation of [5], define a random "state of nature" parameter $\boldsymbol{\theta}$ by $\boldsymbol{\theta} = n$ if h_n is true, $n = 0, 1, ..., N$. A prior distribution for $\boldsymbol{\theta}$ is assumed and a test (i.e., a *Mode A* test or a *Mode B* test on a particular cell $\mathcal{C}_n$) is chosen and performed yielding a binary outcome $\mathbf{b}$. If $\Pr(H_0|\mathbf{b}) > \Pr(H_1|\mathbf{b})$, the system decides for H_0. Otherwise, the system decides in favor of the hypothesis h_n having the largest posterior probability $\Pr(h_n|\mathbf{b})$; i.e., in this case the system decision rule ϕ takes the value n if h_n has the largest posterior probability. As shown in [1], these posterior probabilities are straightforward to compute using the detection and false alarm probabilities of the chosen test, which follow from the (prior) distribution of $\boldsymbol{\theta}$. The overall algorithm is depicted schematically in Figure 1.

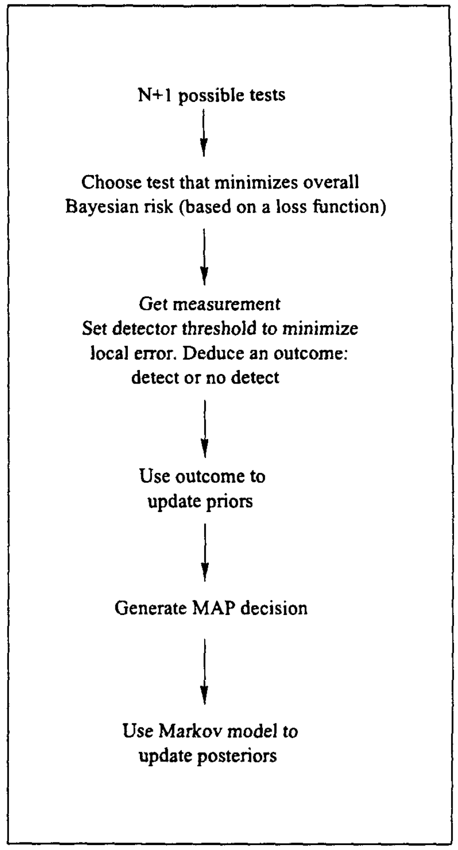

Figure 1: A single test iteration in the sequential detection/localization algorithm for a moving target.

Note that, once a test is selected, the rule ϕ for choosing a hypothesis h_n based on the test's outcome is well defined.

The approach to mode (i.e., test) selection is to choose the one that minimizes Bayes risk with respect to a pre-defined loss functional. Since the overall goal of the system is to both detect the signal source *and* localize it, with these two subgoals possibly being of unequal importance, a loss functional of the following form is used:

$$L(\theta, \phi) = \begin{cases} 0 & \theta = \phi \\ c_0 & \theta \neq \phi,\ \phi \neq 0, \text{ and } \theta \neq 0 \\ c_1 & \theta \neq \phi \text{ and } \theta = 0 \\ c_2 & \theta \neq \phi \text{ and } \phi = 0 \end{cases}$$

With $c_1 > c_0$, this functional imposes a greater penalty for a false alarm (i.e., deciding in favor of H_1 when H_0 is true) than for correct detection with incorrect localization (i.e., H_1 is correctly chosen, but the wrong cell is picked). With $c_2 > c_1$, an even greater penalty is levied if the system decides in favor of H_0 when a target is actually present. Depending on the application, the weights c_1 and c_2 can be chosen to adjust the relative importance of detection and classification in an intuitively appealing way.

With this loss functional, the risk is

- For $\boldsymbol{\theta} = 0$
  $$R(\boldsymbol{\theta}, \phi) = c_1 \sum_{n \neq 0} \Pr(\rightarrow h_n | h_0)$$
- For $\boldsymbol{\theta} \neq 0$
  $$R(\boldsymbol{\theta}, \phi) = c_2 \Pr(\rightarrow h_0 | h_k) + c_0 \sum_{n=1, n \neq k}^{N} \Pr(\rightarrow h_n | h_k)$$

and the Bayes average risk of the decision rule ϕ is thus

$$R(\phi) = \int R(\boldsymbol{\theta}, \phi) dF(\boldsymbol{\theta})$$

$$= c_1 \Pr(h_0) \textstyle\sum_{n=1}^{N} \Pr(\rightarrow h_n | h_0) +$$

$$\sum_{n=1}^{N} \Pr(h_n) \left[c_2 \Pr(\rightarrow h_0 | h_n) + c_0 \sum_{k=1, k \neq n}^{N} \Pr(\rightarrow h_k | h_n) \right]$$

Since the decision rule ϕ (and hence each decision $\rightarrow h_n$) depends on the probabilities of the hypotheses h_k posterior to the test, this quantity depends on which test (mode) is selected.

The mode is chosen to minimize the conditional expectation of the Bayes average risk. The posterior probabilities of each h_n given a particular test and its outcome can be calculated using Bayes' rule, the prior probabilities of the hypotheses, and the detectors' probabilities of detection and false alarm. These calculations are given explicitly in [1]. Once a mode is selected and the test T_m is performed, the decision $\rightarrow h_n$ is completely determined by its outcome $o_m \in \{0,1\}$; prior to performing the test, the only uncertainty about the decision arises because the outcome of the test is not yet known. The conditional expectation of the Bayes average risk for a chosen test T_m is $R_0 \Pr(\mathbf{b}_m = 0) + R_1 \Pr(\mathbf{b}_m = 1)$ where

$$\begin{aligned} R_0 &= \mathsf{E}[R(\boldsymbol{\theta},\phi)|\boldsymbol{\theta},\mathbf{b}_m = 0] \\ &= c_1 \Pr(h_0|\mathbf{b}_m = 0) \sum_{n=1}^{N} \Pr(\rightarrow h_n|h_0, \mathbf{b}_m = 0) \\ &\quad + \sum_{n=1}^{N} \Pr(h_n|\mathbf{b}_m = 0)[c_2 \Pr(\rightarrow h_0|h_n, \mathbf{b}_m = 0) \\ &\quad + c_0 \sum_{k=1, k\neq n}^{N} \Pr(\rightarrow h_k|h_n, \mathbf{b}_m = 0)] \end{aligned}$$

$$\begin{aligned} R_1 &= \mathsf{E}[R(\boldsymbol{\theta},\phi)|\boldsymbol{\theta},\mathbf{b}_m = 1] \\ &= c_1 \Pr(h_0|\mathbf{b}_m = 1) \sum_{n=1}^{N} \Pr(\rightarrow h_n|h_0, \mathbf{b}_m = 1) \\ &\quad + \sum_{n=1}^{N} \Pr(h_n|\mathbf{b}_m = 1)[c_2 \Pr(\rightarrow h_0|h_n, \mathbf{b}_m = 1) \\ &\quad + c_0 \sum_{k=1, k\neq n}^{N} \Pr(\rightarrow h_k|h_n, \mathbf{b}_m = 1)] \end{aligned}$$

The complexity of these expressions for R_0 and R_1 belie their relatively simple nature. There are two cases:

- **Case 1**: $\phi = 0$

$$R_0 = c_2 \Pr(H_1|\mathbf{b}_m = 0)$$

$$R_1 = c_2 \Pr(H_1|\mathbf{b}_m = 1)$$

- **Case 2**: $\phi = k > 0$

$$R_0 = c_1 \Pr(H_0|\mathbf{b}_m = 0) + c_0 \Pr(H_1|\mathbf{b}_m = 0)[\Pr(H_1|\mathbf{b}_m = 0) - \Pr(h_k|\mathbf{b}_m = 0)]$$

$$R_1 = c_1 \Pr(H_0|\mathbf{b}_m = 1) + c_0 \Pr(H_1|\mathbf{b}_m = 1)[\Pr(H_1|\mathbf{b}_m = 1) - \Pr(h_k|\mathbf{b}_m = 1)]$$

The conditional probabilities in these expressions are exactly the post-test probabilities computed in [1]; the probabilities of the test outcomes are computed as follows. For a *Mode A* test,

$$\begin{aligned} \Pr(\mathbf{b}_n = 0) &= (1 - P_{d,A}) \Pr(H_1) + (1 - P_{f,A}) \Pr(H_0) \\ \Pr(\mathbf{b}_n = 1) &= P_{d,A} \Pr(H_1) + P_{f,A} \Pr(H_0) \end{aligned}$$

and for a *Mode B* test,

$$\begin{aligned} \Pr(\mathbf{b}_n = 0) &= (1-P_{d,B}^{(n)}) \Pr(h_n) + (1-P_{f,B}^{(n)})(1-\Pr(h_n)) \\ \Pr(\mathbf{b}_n = 1) &= P_{d,B}^{(n)} \Pr(h_n) + P_{f,B}^{(n)}(1 - \Pr(h_n)) \end{aligned}$$

In these expressions, $P_{d,B}^{(n)}$ and $P_{f,B}^{(n)}$ are the probabilities of detection and false alarm, respectively, of the *Mode B* detector used on cell $\mathcal{C}_n$. $P_{d,A}$ and $P_{f,A}$ are the corresponding probabilities for the *Mode A* detector.

To summarize, the decision rule ϕ depends on the outcome of the test and the posterior probabilities of h_n, $n = 1, ..., N$. These can be computed *before any test is actually run*. Thus, for each candidate test, the expected risk may be calculated using $\Pr(\mathbf{b}_m = 0)$ and $\Pr(\mathbf{b}_m = 1)$ (which come from the detector performance figures) before running any tests. This allows the selection of the test of lowest Bayes risk, as proposed.

The update of the priors (computation of the posterior probabilities) is done as follows. At the outset of this step a test has been performed, either in *Mode A* on all of $\mathcal{C}$ or in *Mode B* on a specific cell $\mathcal{C}_n$. The *a posteriori* probabilities $\Pr(h_n)$ for $n = 0, ..., N$ given the outcome of the test can be evaluated by Bayes' rule. The posterior probabilities are computed on a case-by-case basis in terms of the detectors' probabilities of detection $P_{d,A}$, $P_{d,B}^{(n)}$ and false alarm $P_{f,A}$, $P_{f,B}^{(n)}$ as follows.

- *Mode A*, $\mathbf{b}_n = 1$:

$$\begin{aligned} \Pr(H_1|\mathbf{b}_n = 1) &= \sum_{n=1}^{N} \Pr(h_n|\mathbf{b}_n = 1) \\ \Pr(h_n|\mathbf{b}_n = 1) &= \frac{P_{d,A} \Pr(h_n)}{P_{f,A} p_0 + P_{d,A} p_1} \end{aligned}$$

- *Mode A*, $\mathbf{b}_n = 0$:

$$\begin{aligned} \Pr(H_1|\mathbf{b}_n = 0) &= \sum_{n=1}^{N} \Pr(h_n|\mathbf{b}_n = 0) \\ \Pr(h_n|\mathbf{b}_n = 0) &= \frac{(1 - P_{d,A}) \Pr(h_n)}{(1 - P_{f,A}) p_0 + (1 - P_{d,A}) p_1} \end{aligned}$$

- *Mode B* on cell $\mathcal{C}_n$, $\mathbf{b}_n = 1$:

$$\Pr(H_1|\mathbf{b}_n = 1) = \sum_{k=1}^{N} \Pr(h_k|\mathbf{b}_n = 1)$$

$$\Pr(h_k|\mathbf{b}_n = 1) = \begin{cases} \frac{P_{f,B}^{(n)} \Pr(h_k)}{P_{d,B}^{(n)} \Pr(h_n) + P_{f,B}^{(n)}(1-\Pr(h_n))} & k \neq n \\ \frac{P_{d,B}^{(n)} \Pr(h_n)}{P_{d,B}^{(n)} \Pr(h_n) + P_{f,B}^{(n)}(1-\Pr(h_n))} & k = n \end{cases}$$

- *Mode B* on cell C_n, $\mathbf{b}_n = 0$:

$$\Pr(H_1|\mathbf{b}_n = 0) = \sum_{k=1}^{N} \Pr(h_k|\mathbf{b}_n = 0)$$

$$\Pr(h_k|\mathbf{b}_n = 0) = \begin{cases} \frac{(1-P_{f,B}^{(n)})\Pr(h_k)}{(1-P_{d,B}^{(n)})\Pr(h_n)+(1-P_{f,B}^{(n)})(1-\Pr(h_n))} & k \neq n \\ \frac{(1-P_{d,B}^{(n)})\Pr(h_n)}{(1-P_{d,B}^{(n)})\Pr(h_n)+(1-P_{f,B}^{(n)})(1-\Pr(h_n))} & k = n \end{cases}$$

Motion Model:

As described earlier, the decision ($\rightarrow h_n$) is chosen to maximize the *a posteriori* probability of the target being present in any particular cell. It is assumed that the target is stationary during the time between the measurement and the decision outcome. Between measurements, the target is modeled as a Markov process. In the examples that follow, we considered a particular case where the target is traveling South. As noted in Figure 2, at each iteration step, the target will remain in its current cell with probability q, will move straight down one cell with probability d, will move one cell down and to the right (left) with probability r (l) unless it is on the right border, left border or the South-most end of the region. In the case of the target being on any of the borders, the probability of remaining in the current cell is increased. Once the target reaches the the South-most region, it becomes stationary.

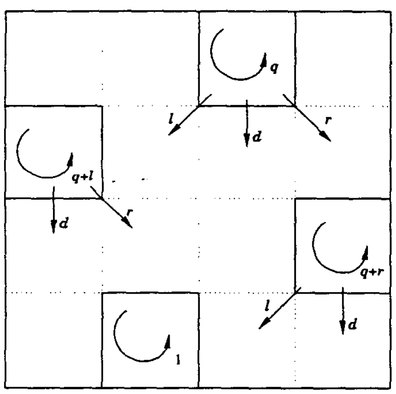

Figure 2: Markov motion model.

It is assumed that the detector has complete knowledge of the target motion model. As a result, the posterior probabilities are updated using the Markov model before they become the priors for the following iteration.

4 Examples

The following two examples show the behavior of the two-mode detection/localization system operating in a 4-by-4 cell scenario. The cells are denoted by $C_{i,j}$ for $i, j = 1, ..., 4$. The motion model assumes the following values for the transition probabilities: $q = 0.5$, $d = 0.3$, and $l = r = 0.1$. Note that $q + d + l + r = 1$. The termination criterion used in this case is any of the individual probabilities h_n exceeding the value 0.9.

Example 1:

The test signal and white Gaussian noise vectors are of length $M = 10$ and the SNRs in the two modes are -3.1 dB and 1.7 dB, respectively. The cost values are $c_0 = 1$, $c_1 = 1.2$, and $c_2 = 2$. The initial prior probabilities are given in Figure 3. The star ($\star$) denotes a target present in cell $C_{1,1}$ at the onset of the search.

$\star$ 0.0056	0.0328	0.0775	0.0179
0.0994	0.0178	0.0434	0.0554
0.0158	0.0947	0.0422	0.0677
0.0406	0.0341	0.0534	0.0017

Figure 3: Prior probabilities (Example 1).

Figure 4 illustrates the iterative search in this example. For each test, the posterior probabilities of the test, which are updated using the Markov model to be used as the prior probabilities in the next test, appear in the 4-by-4 grid. In this example, a signal source is actually present in cell $C_{1,1}$ (indicated by a star in the upper left corner) at the onset of the search. The system chooses *Mode A* for the initial test (indicated by the horizontal lines delimiting both the upper and lower boundary of the geographical area seen by the detector), detects (per the annotation on the header), and decides for cell $C_{2,1}$ (indicated by a triangle in the upper left corner). In test II, the target has moved to cell $C_{2,1}$ and the system runs in *Mode B* on cell $C_{4,3}$, does not detect but decides for cell $C_{4,2}$ because it has the highest posterior (and $p_1 > 0.5$) *before applying the Markov model since no motion is assumed to have occurred yet.* In test III, the target has moved to cell $C_{3,1}$ and the detector runs again in *Mode B* but on cell $C_{4,2}$, does not detect and decides for cell $C_{4,1}$. Finally, in test IV, with the target in cell $C_{4,1}$, the detector runs in *Mode B* on cell $C_{4,1}$, detects and decides for cell $C_{4,1}$. Since $p_1 > 0.9$ (one of the preset criteria for termination), the iterative detection/localization scheme terminates having, in this case, reached the right final decision.

Test I: b = 1

★ 0.0043	0.0212	0.0500	0.0138
▷ 0.0833	0.0349	0.0645	0.0598
0.0530	0.0863	0.0534	0.0794
0.0707	0.0881	0.1061	0.0338

Test II: b = 0

0.0029	0.0117	0.0277	0.0092
★ 0.0591	0.0324	0.0562	0.0498
0.0668	0.0758	0.0615	0.0798
0.1054	▷ 0.1380	0.0464	0.0698

Test III: b = 0

0.0020	0.0067	0.0159	0.0063
0.0430	0.0261	0.0441	0.0406
★ 0.0700	0.0678	0.0640	0.0784
▷ 0.1525	0.0527	0.0921	0.1144

Test IV: b = 1

0.0001	0.0002	0.0005	0.0002
0.0016	0.0010	0.0017	0.0017
0.0035	0.0030	0.0031	0.0038
▷ ★ 0.9504	0.0052	0.0076	0.0087

Figure 4: Test sequence (Example 1).

Example 2:

In this second example, the test signal and white Gaussian noise vectors are also of length $M = 10$, but the SNRs in the two modes are -4.4 dB and 7.6 dB (i.e., the difference in *Mode A* and *Mode B* detector performance is much greater than in the preceding example). The cost values are still $c_0 = 1$, $c_1 = 1.2$, and $c_2 = 2$. The initial prior probabilities are given in Figure 5. A target is present in cell $C_{2,1}$ at the onset of the search.

0.0536	0.0663	0.0372	0.0597
★ 0.0242	0.0395	0.0751	0.0744
0.0177	0.0827	0.0466	0.0017
0.0014	0.0414	0.0180	0.0605

Figure 5: Prior probabilities (Example 2).

Figure 6 illustrates the evolution of the iterative search. In test I, the detector operates in *Mode A*, detects, and chooses cell $C_{3,2}$. Then, the detector runs in *Mode B* on cell $C_{4,2}$, does not detect and chooses cell $C_{2,4}$. Note that the target was still in cell $C_{2,1}$. In test III, with the target having moved to cell $C_{3,1}$, the detector runs again in *Mode B* but on cell $C_{4,4}$, does not detect and chooses cell $C_{4,3}$. With the target remaining in cell $C_{3,1}$, the detector runs in *Mode B* on cell $C_{4,3}$, does not yield a detection and opts for cell $C_{4,2}$. By test V, the target is in cell $C_{4,1}$, the detector runs on cell $C_{4,2}$ and decides in favor of the right cell, namely $C_{4,1}$. In the final iteration, the detector runs in *Mode B* on cell $C_{4,1}$, detects, and chooses, in this second example as well, the cell with the target.

5 Conclusion and Future Work

A Bayesian approach for optimal management of a switchable-mode detection system has been developed. Although attention was focused on the two-mode case, the principles used should extend directly to cases involving more modes.

The examples were terminated when posterior probability of some cell (or h_0) exceeded a threshold, with correct decisions on both signal presence and location. In other trials, the system occasionally did not attain this termination condition in a reasonable number of iterations, suggesting the need for convergence analysis of the iterative algorithm and attention to development of appropriate termination criteria.

As noted, one case of particular interest is where it is known *a priori* that a fixed number of iterations

Figure 6: Test sequence (Example 2).

Test I: $\mathbf{b} = 1$

0.0402	0.0414	0.0232	0.0447
★ 0.0465	0.0609	0.0766	0.0828
0.0272	▷ 0.0789	0.0715	0.0386
0.0186	0.0907	0.0505	0.0820

Test II: $\mathbf{b} = 0$

0.0265	0.0228	0.0128	0.0295
★ 0.0485	0.0541	0.0593	▷ 0.0719
0.0400	0.0770	0.0804	0.0612
0.0382	$\underline{\overline{0.0369}}$	0.0921	0.1108

Test III: $\mathbf{b} = 0$

0.0179	0.0128	0.0072	0.0199
0.0442	0.0425	0.0435	0.0599
★ 0.0494	0.0737	0.0794	0.0722
0.0651	0.0810	▷ 0.1462	$\underline{\overline{0.0297}}$

Test IV: $\mathbf{b} = 0$

0.0126	0.0075	0.0042	0.0140
0.0389	0.0323	0.0318	0.0500
★ 0.0553	0.0684	0.0738	0.0769
0.1022	▷ 0.1359	$\underline{\overline{0.0450}}$	0.0695

Test V: $\mathbf{b} = 0$

0.0087	0.0043	0.0024	0.0097
0.0322	0.0233	0.0224	0.0400
0.0556	0.0590	0.0633	0.0744
▷ ★ 0.1454	$\underline{\overline{0.0387}}$	0.0945	0.1156

Test VI: $\mathbf{b} = 1$

0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
▷ ★ $\underline{\overline{0.9998}}$	0.0000	0.0000	0.0000

will be used. It would appear to be possible, in principle, to evaluate the propagation of the hypothesis probabilities exhaustively to the last iteration — considering every possible combination of possible tests and outcomes — *before actually performing any of the tests.* This would allow selection of an optimal sensing strategy at the expense of a substantial computational burden. Efficient algorithms for attaining near-optimal sensing strategies in such cases would be of significant interest.

Another avenue for further research is the scenario involving multiple hypotheses and multiple modalities. This generalization provides a natural mechanism to address signal classification problems; it is under current investigation. It is important to note that, especially when the number of cells becomes large, it would be reasonable to assume that the cells are statistically dependent. With correlated cells, the algorithm is expected to perform better.

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