TRANSIENT SIGNAL DETECTION USING UNITARY TRANSFORMATIONS

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ABSTRACT

This paper examines the application of axis warping functions, a class of unitary transformation on the space of finite-energy signals, in detection of transient signals. Many signals associated with rotating machinery subjected to time-varying mechanical loading, speed changes, or motion relative to a sensor are well modeled by sinusoids transformed by this type of warping. This observation provides a new perspective on detection of an important class of transient signals.

1. INTRODUCTION

The use of unitary operators to generate new classes of signal analysis functions has been proposed and studied in recent literature. Based on the observation that there exist large classes of signals for which neither constant-bandwidth (e.g., short-time Fourier transform) nor proportional-bandwidth (e.g., wavelet) analysis is appropriate, new classes of signal representations and analysis systems that generalize the concepts of time, frequency, and scale have been derived by applying various unitary transformations to well-established sets of analysis functions [1, 2, 3, 4, 5]. There has also been interest in the related topic of developing generalized joint signal representations [8, 10, 12], time-frequency representations, and time-scale representations, some of this in connection with detection in nonstationary settings [8, 9, 11, 13, 14, 15].

The paper focuses on a particular class of unitary transformations known as axis warpings. Mathematical models of rotating machinery under time-varying mechanical loading, speed changes, and motion relative to a sensor are shown to give rise to signals that are of the form of sinusoids subjected to axis warping transformations. This perspective allows physical knowledge about the signal source to be directly coupled into a signal model which, in turn, yields a unitary transformation on the standard Fourier basis that is matched to the signal. Processing with the warped Fourier basis concentrates the energy of the signal into a single coefficient. A close connection between this procedure and matched filtering with a signal replica produced by the mathematical model of the underlying mechanical system is noted.

The concept of unitary axis warping is described in Section 2. The idea behind detection of warped sinusoids is presented in Section 3. Applications of the proposed technique to both real and simulated data are presented in Section 4. Sections 5 and 6 contain discussions of results and conclusions respectively.

2. UNITARY AXIS WARPING

A unitary operator U on $L^2(\mathbb{R})$ is a linear, surjective transformation that preserves inner product; i.e.,

$$\langle Uf, Ug \rangle = \langle f, g \rangle \tag{1}$$

where f and g are functions in $L^2(\mathbb{R})$. Orthonormal bases are mapped to orthonormal bases by a unitary transformation; i.e., if $\{b_k | k \in \mathbb{Z}\}$ is an orthonormal basis for L^2, then $\{Ub_k | k \in \mathbb{Z}\}$ is also an orthonormal basis for L^2. The adjoint U^* of an operator U is defined by

$$\langle U^* f, g \rangle = \langle f, Ug \rangle \tag{2}$$

In the case of unitary operators, this clearly implies

$$U^* U = I \tag{3}$$

i.e.,

$$U^*[Uf] = f \tag{4}$$

An important subclass of unitary transformations on $L^2(\mathbb{R})$ is the axis warping of the following form:

$$[W_\gamma f](t) = \sqrt{|\gamma'(t)|}\, f[\gamma(t)] \tag{5}$$

where $\gamma(t)$ is a smooth, monotonic function. It is a straightforward calculation to verify that W_γ is unitary.

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The adjoint of W_γ (axis unwarping operator) is given by

$$[W_\gamma^* g](t) = \sqrt{|[\gamma^{-1}]'(t)|}\; g[\gamma^{-1}(t)] \tag{6}$$

To verify this, let $g(t) = [W_\gamma f](t) = \sqrt{|\gamma'(t)|}\; f[\gamma(t)]$. Then

$$\begin{aligned}[W_\gamma^* g](t) &= \sqrt{|[\gamma^{-1}]'(t)|}\; g[\gamma^{-1}(t)] \\ &= \sqrt{|[\gamma^{-1}]'(t)|}\; \sqrt{|\gamma'[\gamma^{-1}(t)]|}\; f\{\gamma[\gamma^{-1}(t)]\} \\ &= f(t)\end{aligned} \tag{7}$$

since $[\gamma^{-1}]'(t) = 1/\gamma'[\gamma^{-1}(t)]$. Thus unwarping of the axis warped signal can be done by warping back the warped signal using $\gamma^{-1}(t)$.

3. DETECTION PRINCIPLES FOR WARPED SINUSOIDS

Many "transient" signal events produced by start-up, shut-down, other speed changes, or Doppler-induced effects on narrowband components of the acoustic signals produced by rotating machinery can be modeled as axis warpings of sinusoidal signals. If the warping function is known, a natural basis for analysis of such signals is produced by warping of the standard Fourier basis. Indeed, if the warping function is known exactly, a warped sinusoid provides a "matched filter" for the transient event.

An important duality in signal processing with unitary operators is that warping a basis set to match a signal is equivalent to prewarping the signal to match an existing basis; i.e.,

$$\begin{aligned}\langle f, W_\gamma b_k\rangle &= \int_{\mathbb{R}} f(t)\overline{\sqrt{\gamma'(t)}b_k(\gamma(t))}dt \\ &= \int_{\mathbb{R}} f(\gamma^{-1}(u))\frac{1}{\sqrt{\gamma'(t)}}\overline{b_k(u)}du \\ &= \int_{\mathbb{R}} \sqrt{[\gamma^{-1}]'(u)}f(\gamma^{-1}(u))\overline{b_k(u)}du \\ &= \langle W_{\gamma^{-1}} f, b_k\rangle\end{aligned} \tag{8}$$

If the signal of interest is a sinusoid subjected to a known warping, this duality implies that analysis of the data with an orthonormal basis of suitably warped sinusoids is equivalent to applying the inverse warping to the data and then analyzing this prewarped data with an orthonormal basis of sinusoids; i.e., standard Fourier analysis. This paper emphasizes the construction of axis warping functions based on mathematical models of mechanical or other physical aspects of the system.

4. EXAMPLES

Two real-data examples are presented in this section: spin-up of an electric motor (vacuum cleaner) and closest point approach (CPA) of a constant-velocity vehicle producing a sinusoidal acoustic signal. Theoretical development, simulation, and actual data analysis are included.

4.1. Spin-Up Model

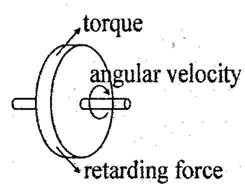


Figure 1: Simple mechanical model of a rotating machine.

Figure 1 shows a simple mechanical model of a rotating machine. Assuming the retarding force is proportional to angular velocity, this system can be modeled by the first-order differential equation

$$\tau(t) - a\,\omega(t) = I\,\frac{d\omega(t)}{dt} \tag{9}$$

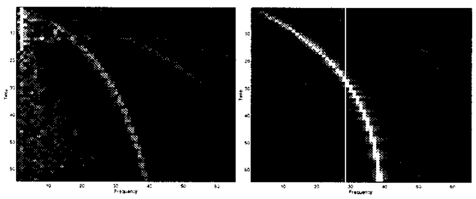
where τ is the applied torque, ω is angular velocity, I is the moment of inertia, and a is a constant. For a given external torque, the angular velocity as a function of time, $\omega(t)$, can be found analytically and the dominant narrowband component of the signal generated by this system can be represented as

$$\sin[\omega(t)t] = \sin[\gamma(t)] \tag{10}$$

where $\gamma(t) = \omega(t)t$ is the axis warping function used for prewarping of the observed signal from this system. For unit step external torque, $\omega(t)$ becomes

$$\omega(t) = \frac{1}{a}\left[1 - e^{-\frac{a}{I}t}\right]u(t) \tag{11}$$

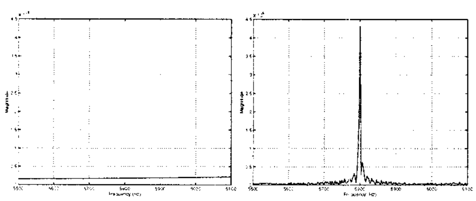
Thus for a unit step external torque applied to this system, angular velocity increases exponentially going to the steady state as a function of time. This model is used for spin-up of an electric motor. Simulated data is compared with actual data. Figure 2 (a) shows a time-frequency plot of the real vacuum data. Simulated data is generated by warping a single sinusoid with $\gamma(t) = c(1 - e^{-\alpha t})t$ where c is the constant for the steady state frequency value and α is the constant for rate of exponential frequency increase. These two variables, c and α, are chosen such that the time-frequency plot of the generated data is as nearly identical to that of the actual vacuum data as possible. Figure 2 (b) shows a time-frequency plot of the generated simulated data.



(a) (b)

Figure 2: Spectrograms of (a) real vacuum data and (b) simulated vacuum data. Time increases downward on the vertical axis and frequency is on the horizontal axis.

The results of proposed analysis technique on both simulated and real vacuum data are shown in Figure 3. Figures 3 (a) and (c) show power spectral estimates from the original data; (b) and (d) show power spectral estimates from the prewarped data. The same warping function $\gamma(t)$ used to generate simulated data is applied for prewarping in both cases. The result clearly shows energy concentration of the signal considered in this example through the proposed technique. This is expected to correspond to high detectability.



(a) (b) (c) (d)

Figure 3: Power spectra of (a) simulated vacuum data, (b) unwarped simulated vacuum data, (c) real vacuum data, and (d) unwarped real vacuum data.

4.2. Closest Point Approach (CPA) Model

Figure 4 shows a model of CPA, where v is the constant velocity of the vehicle, d is the distance from the sen-

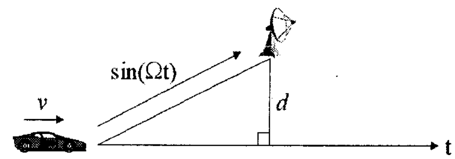


Figure 4: Simple CPA model.

sor to the vehicle at their closest point, c is the speed of sound, and Ω is the constant frequency of the signal source. The Doppler shift at the receiver can be modeled by the following equation

$$\Delta\Omega(t) = -\Omega\frac{v}{c}\frac{vt}{\sqrt{v^2t^2+d^2}} \tag{12}$$

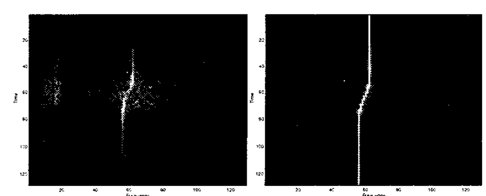
The complete signal at the receiver becomes

$$e^{i(\Omega+\Delta\Omega(t))t} \tag{13}$$

Thus axis warping function for this model becomes the following

$$\gamma(t) = \Omega\left(1-\frac{v}{c}\frac{vt}{\sqrt{v^2t^2+d^2}}\right)t \tag{14}$$

Figure 5 (a) shows a time-frequency plot of the real CPA data. Simulated data was generated by warping a single sinusoid with $\gamma(t)$ given in Equation 14. Variables in Equation 14 are chosen such that the time-frequency plot of the generated data is as nearly identical to that of the actual CPA data as possible. Figure 5 (b) shows a time-frequency plot of the simulated CPA data.



(a) (b)

Figure 5: Spectrograms of (a) real CPA data and (b) simulated CPA data. Axes are as in Figure 2.

The results of proposed analysis technique on both simulated and real CPA data are shown in Figure 6. Figures 6 (a) and (c) show power spectral estimates from the original data; (b) and (d) show power spectral estimates from the prewarped data. The same warping function $\gamma(t)$ used to generate simulated data is applied for prewarping in both cases. Again, the result clearly shows the energy concentration of the signal considered in this example through the proposed technique.

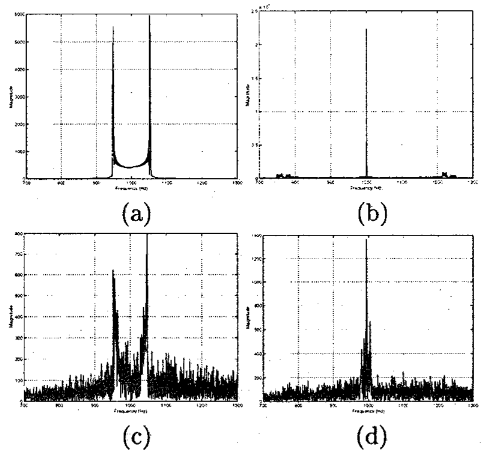


Figure 6: Power spectra of (a) simulated CPA data, (b) unwarped simulated CPA data, (c) real CPA data, and (d) unwarped real CPA data.

5. DISCUSSION

As mentioned in Section 4, variables in the axis warping function $\gamma(t)$ for each example are chosen such that the time-frequency plot of the generated data is as nearly identical to that of the actual data as possible. In this paper, this selection of parameters was done manually. In practice, however, this needs to be done automatically. For mechanical models involving only one or two parameters, it may be possible to use an exhaustive search over the viable range of parameter values. Several possibilities exist for efficiently examining a parameter space that is too large to search exhaustively. Some examples include gradient methods, genetic algorithms, and simulated annealing — using a measure of energy concentration in the spectrum of the unwarped signal as the objective function in each case. Such approaches will be examined in future research.

Figure 7 shows power spectra of unwarped simulated vacuum data using different values of α and Figure 8 shows power spectra of unwarped real vacuum data using different values of α. It is clear from Figure 7 that the right choice of α produces better energy concentration of the unwarped simulated vacuum data which corresponds to higher detectability. However, unlike the simulation results in Figure 7, the change of energy concentration of unwarped real vacuum data is shown to be less sensitive to different values of α which might be more desirable in real life detection scenario. Once the proposed technique decides that there exists a transient signal, one can extract valuable information from estimates of the warping function parameters; e.g., the distance from the sensor to the moving object, the speed of the vehicle, and/or (with some prior knowledge about different vehicles) what kind of vehicle was used in a CPA setting.

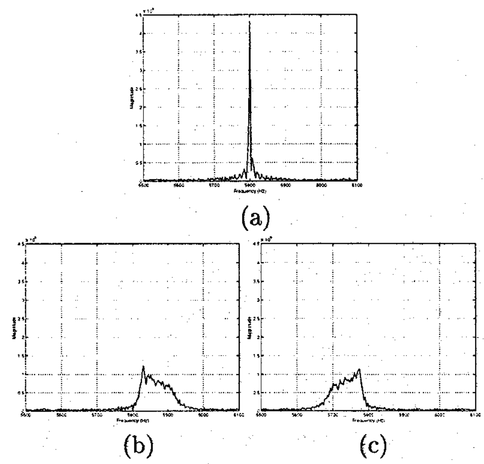


Figure 7: Power spectra of unwarped simulated vacuum data using (a) $\alpha = 4.2$, (b) $\alpha = 4.1$, and (c) $\alpha = 4.3$ without any noise.

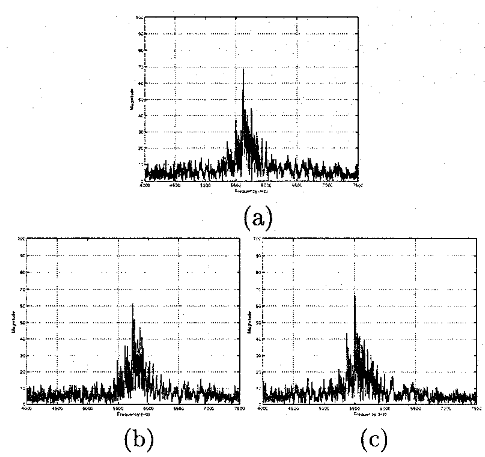


Figure 8: Power spectra of unwarped real vacuum data using (a) $\alpha = 4.2$, (b) $\alpha = 4.0$, and (c) $\alpha = 4.4$

The detection statistic in a matched filter detector is the inner product between the known signal replica and the data to be tested. Since inner products are invariant under unitary transformations, the matched filter statistic for a warped sinusoid may be obtained by standard Fourier analysis of the prewarped data. From the fact that the proposed technique concentrates the energy of the unwarped signal to a fewer frequency components, detection statistic can be formed by comparing the energy of the signal around main frequency component to the rest of the signal energy. This provides the basis for a detector.

6. CONCLUSIONS

Axis warping functions, a class of unitary transformations on L^2, are constructed based on the knowledge about the mechanical or other physical aspects of the system producing a signal to match time-frequency characteristics of transient signal generated by the source. Since many practical transient signals can be modeled by sinusoids subjected to this type of warping, detection is performed by prewarping (unwarping) the signal with the axis warping function found from the mechanical system and taking simple FFT of the unwarped signal. In this paper, both simulation and real-life examples were provided to demonstrate the performance of the proposed method. The detection statistic for matched filtering was also discussed.

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