Non-parametric Multiple Channel Detection in Deep Ocean Noise

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Abstract

The generalized coherence (GC) estimate has recently been studied as a statistic for detection of a common signal on several noisy channels. Although performance results have been documented for a white gaussian signal in white gaussian noise, no meaningful performance measurements have been presented for GC-based detection in colored noise. This paper examines the application of GC detectors in the presence of colored noise and, in particular, gives performance results for a GC detector in one of the prototypical scenarios for non-parametric multiple channel detection: detecting a common broadband signal in multiple channels of deep ocean noise.

1. Introduction

The generalized coherence (GC) estimate has been studied as a statistic for detection of a common signal on $M \geq 2$ noisy channels [1, 3]. For brevity, the notation of [1] is adopted for the remainder of the paper. Given M complex N-vectors $\mathbf{x}_1, ..., \mathbf{x}_M$, the GC estimate is defined as

$$\hat{\gamma}^2_{M,N}(\mathbf{x}_1, ..., \mathbf{x}_M) \mathrel{\hat{=}} 1 - \frac{g(\mathbf{x}_1, ..., \mathbf{x}_M)}{||\mathbf{x}_1||^2 \cdots ||\mathbf{x}_M||^2}$$

where

$$g(\mathbf{x}_1, ..., \mathbf{x}_M) \mathrel{\hat{=}} \det \begin{bmatrix} \langle \mathbf{x}_1, \mathbf{x}_1 \rangle & \dots & \langle \mathbf{x}_1, \mathbf{x}_M \rangle \\ \vdots & \ddots & \vdots \\ \langle \mathbf{x}_M, \mathbf{x}_1 \rangle & \dots & \langle \mathbf{x}_M, \mathbf{x}_M \rangle \end{bmatrix}$$

denotes the determinant of $M \times M$ Gram matrix formed from the sequences $\mathbf{x}_1, ..., \mathbf{x}_M$. Properties of Gram matrix determinants are discussed in [2].

The GC estimate has been shown to provide a natural geometrical generalization of the magnitude-squared coherence (MSC) estimate, a widely used statistic for detection of a common signal on two noisy channels [4]. Recently it has been observed that the GC estimate arises as the test statistic in the uniformly most powerful invariant matched subspace detector for a class of multiple channel detection problems [5, 6].

Explicit knowledge of the GC estimate's probability distribution under the H_0 hypothesis that all M channels contain independent white gaussian noise makes it possible to calculate detection thresholds corresponding to given probabilities of false alarm. Such thresholds are calculated [1] and are then used to develop performance results for detection of a white gaussian signal in two or three channels of white gaussian noise. Use of the GC estimate in scenarios involving colored noise is not addressed, however. This is an important consideration; among the many practical problems involving multiple channel detection in colored noise is detection of a common signal in several channels of deep ocean noise — a motivating problem for much of the published work on coherence-based detection.

This paper proposes an approach for GC-based multiple channel detection in colored noise and evaluates its performance in a deep ocean noise scenario.

2. Approach

The GC-based detector developed in [1] is a significance test. It does not assume an explicit signal model, but relies on the ability of the GC estimate to discern deviations from H_0. If such a deviation occurs due to the addition of a common signal to each channel, the geometrical perspective presented in [1] suggests that the GC estimate will reflect the resultant "clustering" of the data sequences in signal space.

Suppose the M-channel detector is to operate in a scenario

$$\begin{aligned} H_0 &: \mathbf{x}_k(\cdot) = \mathbf{n}_k(\cdot) \\ H_1 &: \mathbf{x}_k(\cdot) = \mathbf{s}(\cdot) + \mathbf{n}_k(\cdot) \end{aligned}$$

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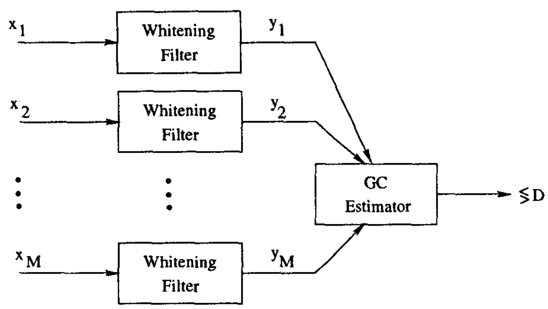

Figure 1. Data $x_k(\cdot)$ from each channel are put through a whitening filter for the colored noise $n_k(\cdot)$. The detection threshold D derived for the white noise case is then directly applicable.

where the noise $\mathbf{n}_k(\cdot)$, $k = 1, ..., M$ on each channel is independent and gaussian with spectral density $S_{\mathbf{n}}(\cdot)$. Under H_0, application of a linear whitening filter for $S_{\mathbf{n}}(\cdot)$ to each channel yields M independent channels of white gaussian noise — precisely the H_0 hypothesis under which detection thresholds were derived in [1]. Linearity of the whitening filter implies that identical signal components present in each channel before filtering remain identical after filtering and thus the "clustering" measured by the GC estimate will still occur unless the signal is completely attenuated by the filter. This reasoning suggests the approach depicted in figure 1. While this approach does not necessarily require that the whitening filters on every channel be the same, use of different filters means that a common additive signal component present on every channel will be distorted differently by each filter prior to coherence estimation.

In this paper, it is assumed that the colored noise can be modeled as a finite-order moving average (MA) process. To obtain performance results, the spectral density of deep ocean noise was approximated by a MA model of order four. The coefficients of the MA filter are derived by spectral factorization (e.g., [9]) of the spectral density depicted on page 209 of [8] in the frequency band from 0 to 256 Hertz. Figure 2 shows the spectral density of the simulated deep sea ocean noise compared to the spectral density from [8]. Estimation of the MA parameters needed for a whitening filter requires solving a system of nonlinear equations in the autocorrelation estimates made from the data. An algorithm described in [7] that employs an autoregressive (AR) approximation for the MA process is used in the implementation presented in this paper because it requires solution of only a linear system of equations.

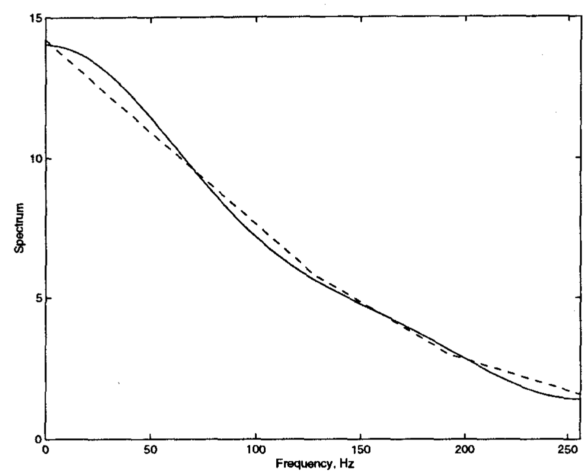

Figure 2. Comparison of actual (dashed line) to simulated (solid line) spectral density of deep ocean noise in the frequency band from 0 to 256 Hertz. Values are multiplied by 10^{-6}.

A whitening filter for the simulated ocean noise was obtained directly from the data by estimating the parameters of an approximating AR process of order $q = 20$. The order of the generating MA filter was not assumed to be known.

3. Effective SNR

In this section the signal-to-noise ratio (SNR) at the output of the whitening filter is derived. For the remainder of this paper it will be called "effective SNR." In general, the effective SNR will be different from the SNR before the whitening filter is applied. The effective SNR is of importance to compare the detection performance of the GC estimate in colored noise to its performance in white noise as documented in [1]. Since the distribution of the GC estimate is not known under H_1 assumptions, it is necessary to evaluate the detection performance of the estimate by computer simulations.

The SNR at the output of the whitening filter can be calculated [9] as

$$\begin{aligned} \mathrm{SNR}_{\mathbf{y}} &= \frac{\operatorname{var}(\mathbf{s}_{\mathbf{y}})}{\operatorname{var}(\mathbf{n}_{\mathbf{y}})} = \frac{C_{\mathbf{s}_{\mathbf{y}}}(0)}{C_{\mathbf{n}_{\mathbf{y}}}(0)} \\ &= \frac{\sum_k \sum_i h(-k) C_{\mathbf{s}_{\mathbf{x}}}(k-i) h^*(-i)}{\sum_k \sum_i h(-k) C_{\mathbf{n}_{\mathbf{x}}}(k-i) h^*(-i)} \end{aligned}$$

where $\mathbf{s}_\mathbf{x}(\cdot)$ is the input signal, $h(\cdot)$ is the impulse response of the whitening filter, and $C_\mathbf{s}(\cdot)$, $C_\mathbf{n}(\cdot)$ denote the autocovariance functions of $\mathbf{s}$ and $\mathbf{n}$, respectively.

For the case of a white gaussian signal in white gaussian noise the whitening filter is the identity and

$$\mathrm{SNR}_\mathbf{y} = \mathrm{SNR}_\mathbf{x}$$

Results from computer simulations, which are presented in the next section, support the conjecture that the detection performance of the GC estimate in white gaussian noise does not depend on the shape of the spectral density of the signal but only on the effective SNR. From this it follows that if the spectral density of the noise is known and an ideal whitening filter is available then the performance of the GC estimate in colored noise is as shown in the receiver operating characteristic (ROC) curves in [1]. At this point, only empirical evidence is available to justify this conjecture.

4. Simulations

Computer simulations based on the approach described above were undertaken to (i) verify that the detection thresholds provide the desired false alarm probabilities, and (ii) test the performance of the GC estimate for detecting a common broadband signal component in multiple channels of colored noise. All simulations were restricted to the three channel case.

4.1 Validation of detection thresholds

The first simulation establishes that detection thresholds derived to provide desired false alarm probabilities in the white noise case remain accurate when used in the pre-whitening approach described above. The results of a Monte Carlo simulation are shown in table 1 for three cases. For a given probability of false alarm, the table contains the probability of false alarm obtained from Monte Carlo simulations for

1. $\hat{\gamma}_w^2(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$: white signal in white noise

2. $\hat{\gamma}_{c_1}^2(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$: white signal in colored noise of known spectral density

3. $\hat{\gamma}_{c_2}^2(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$: white signal in colored noise where the spectral density of the noise must be estimated

Table 1 verifies that the detection thresholds remain valid in the colored noise case.

	White Noise	Colored Noise	
P_{fa}	$\hat{\gamma}_w^2(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$	$\hat{\gamma}_{c_1}^2(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$	$\hat{\gamma}_{c_2}^2(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$
10^{-1}	0.10053	0.10046	0.10031
10^{-2}	0.01071	0.01063	0.01088
10^{-3}	0.00116	0.00116	0.00111
10^{-4}	0.00009	0.00009	0.00007
10^{-5}	0	0	0
10^{-6}	0	0	0

Table 1. Verification of detection thresholds for a three channel GC estimate with $N = 128$ in simulated deep ocean noise.

4.2 Performance evaluation

The second set of simulations obtains performance measurements for three-channel GC-based detection in colored noise. Presented are the results of Monte Carlo simulations for 1000 trials. A three channel GC estimate with $M = 128$ samples was used.

In figure 3, the performance of a GC-based detector for detecting a white signal in simulated deep ocean noise is compared to its performance for detecting the same signal in white noise. The SNR before the whitening filter is fixed to -6dB. The graph shows the performance in two cases: (i) pure noise is available to obtain parameters for the whitening filter and (ii) filter parameters have to be estimated from combined signal and noise data. Note that the detector performs better in colored noise than in white noise. The reason is that the whitening filter increases the SNR by amplifying the portion of the frequency spectrum where the noise level is lowest (and hence the SNR is highest, since the signal is white) and attenuating the portion of the frequency spectrum where the noise level is highest. As expected, the performance is better if the spectral shape of the noise is known than if it needs to be estimated from combined signal and noise data.

In figure 4, the performance of the estimate for a white signal in white noise is compared to the performance of a white signal in colored noise with known spectral density for the same effective SNR of -6dB. Detector performance in these two cases is approximately equal. Since the white signal is colored by the whitening filter, this simulation supports the conjecture that the performance does not depend on the spectral density of the (gaussian) signal. Other simulations with signals of different coloration yielded similar results.

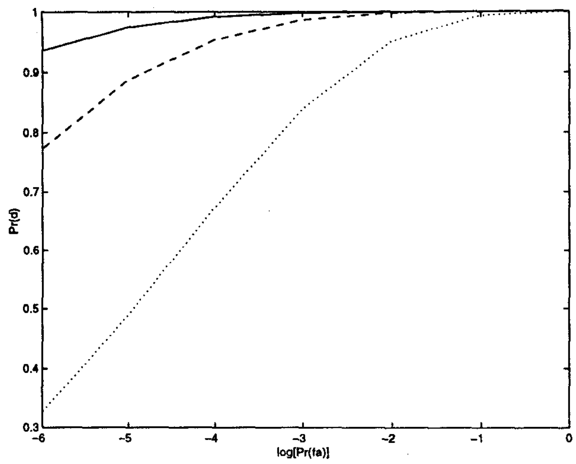

Figure 3. Comparison of the performance of the three-channel GC estimate with a white gaussian signal at SNR=-6 dB on all channels. The dotted line shows the performance of the GC estimate in white noise, the solid line shows the performance in simulated deep ocean noise when pure noise samples were used to estimate the parameters of the whitening filter. The dashed line shows performance where parameters of the whitening filter had to be estimated with signal present.

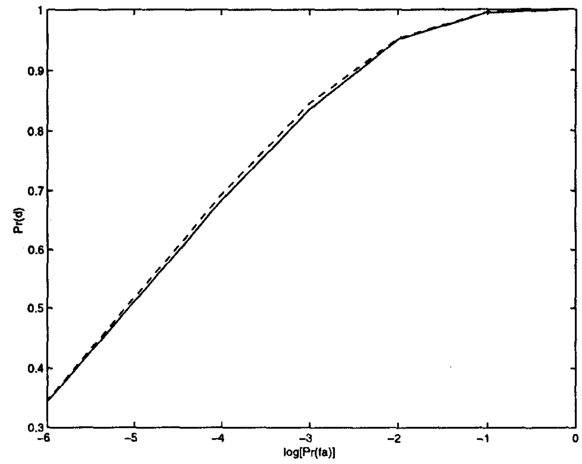

Figure 4. Comparison of the receiver operating characteristic for white signal in white noise (solid line) against white signal in colored noise (dashed line) where the spectral shape of the noise is known for same effective SNR = -6dB at the input of the GC estimate.

5. Conclusions

An approach for use of the GC estimate as a detection statistic in a colored noise environment has been proposed and its utility has been verified empirically using gaussian noise having the spectral density of deep ocean noise. The whitening approach guarantees that, in the noise only case, the distribution of the estimate (and therefore the detection thresholds) derived for use in white gaussian noise remain accurate. As expected, the performance decreases slightly if the spectral density of the noise is not known and parameters of noise process need to be estimated.

Results from simulations lead us to the conjecture that the performance of the GC estimate in white gaussian noise does not depend on the spectral density of the signal, but only on the effective SNR at the input of the estimate.

If this is true, a quantifiable tradeoff between SNR and data sequence length N will exist. In practice, N is the product of the sampling rate (processing bandwidth) and integration time. If the SNR is constant across a broad spectrum, as in the case of a white signal in white noise, increasing the processing bandwidth will increase N and thus improve detection performance. Similarly, if the SNR is constant over a long time period, as in the case of a stationary signal in stationary noise, detector performance can be improved by increasing the integration time. If, however, one is faced with a situation in which the processing bandwidth or integration time can only be increased at the expense of the overall effective SNR, the question of how much SNR loss is compensated by the increase in N. Investigation of the conjecture, both empirically and theoretically is ongoing; the tradeoff question just posed has not been examined as yet.

As a closing remark, the authors note that they are aware of ongoing work investigating the detection performance of the GC estimate for non-gaussian signals in gaussian noise and for signals in multiplicative noise.

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