A Geometric Approach to Multiple-Channel Signal Detection

Douglas Cochran, *Member, IEEE,* Herbert Gish, *Member, IEEE,* and Dana Sinno, *Student Member, IEEE*

***Abstract*— This paper introduces the generalized coherence (GC) estimate and examines its application as a statistic for detecting the presence of a common but unknown signal on several noisy channels. The GC estimate is developed as a natural generalization of the magnitude-squared coherence (MSC) estimate—a widely used statistic for nonparametric detection of a common signal on two noisy channels.**

The geometrical nature of the GC estimate is exploited to derive its distribution under the H_0 hypothesis that the data channels contain independent white Gaussian noise sequences. Detection thresholds corresponding to a range of false alarm probabilities are calculated from this distribution. The relationship of the H_0 distribution of the GC estimate to that of the determinant of a complex Wishart-distributed matrix is noted.

The detection performance of the three-channel GC estimate is evaluated by simulation using a white Gaussian signal sequence in white Gaussian noise. Its performance is compared with that of the multiple coherence (MC) estimate, another nonparametric multiple-channel detection statistic. The GC approach is found to provide better detection performance than the MC approach in terms of the minimum signal-to-noise ratio on all data channels necessary to achieve desired combinations of detection and false alarm probabilities.

I. Introduction

The ability to detect the presence of a common but unknown signal on two or more noisy data channels is desirable in a variety of applications, particularly in situations where an unknown signal source is to be detected and located using data received at two or more spatially distributed sensors. These situations occur in geological settings, such as locating the epicenter of an earthquake using seismic data received at several geophones. They are also encountered in noninvasive testing of machinery [1], [2] and in certain biomedical diagnostic sensing applications. The majority of existing research on nonparametric multiple-channel signal detection has been motivated by applications in passive sonar and radar, however.

In the case of two channels, a popular test for the presence of a common signal component is comparison of the magnitude-squared coherence (MSC) estimate computed using sample sequences from the two channels to a threshold [3]. This is an especially useful test because the statistical behavior of the MSC estimate is well understood in the absence of a common signal, thus allowing thresholds corresponding to particular false alarm probabilities to be readily established.

Manuscript received April 6, 1993; revised September 6, 1994. This work was supported by Arizona State University under FGIA No. 90-127 and RIA No. 90-0733. The associate editor coordinating the review of this paper and approving it for publication was Prof. Isabel Lourtie.

D. Cochran is with the Department of Electrical Engineering, Arizona State University, Tempe, AZ 85287-5706 USA.

H. Gish is with Bolt, Beranek, and Newman Inc., Cambridge, MA 02138 USA.

D. Sinno is with the Telecommunications Research Center, Arizona State University, Tempe, AZ 85287-7206 USA.

IEEE Log Number 9413321.

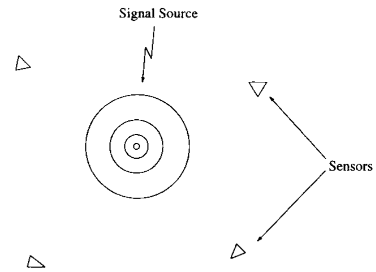


Fig. 1. In a typical scenario, the vectors $\mathbf{x}_m$ arise from sampling the outputs of M geographically distributed sensors. Bandpass filtering to the band of interest and compensation for propagation delays and doppler effects are assumed to have been performed prior to formation of the vectors.

This paper describes an approach for detection of a common signal on M noisy channels that generalizes the MSC approach in a natural way. Specifically, the MSC estimate was shown in [4] to have a geometrical interpretation in light of which its statistical properties can be derived by direct and intuitive means. This geometrical perspective suggests the generalized coherence (GC) estimate, a test statistic for M channels based on the normalized determinant of a $M \times M$ *Gram matrix.* A more precise mathematical formulation of the the multiple-channel detection problem addressed using the GC estimate is presented in the following section. In the third section, the MSC estimate is defined, and its properties are discussed in the context of the two-channel nonparametric detection problem. The distribution function of the MSC estimate is then derived under explicit signal-absent (H_0) assumptions by formulating it in terms of the determinant of a 2×2 Gram matrix. The next section defines the M-channel GC estimate in terms of the determinant of a $M \times M$ Gram matrix, discusses its properties, and derives its H_0 distribution. Subsequent sections present performance results for a GC detector and compare these results to those obtained using another nonparametric multiple channel detection statistic (the multiple coherence estimate).

II. Problem Formulation

Consider M complex random processes $r_1, \ldots, r_M$ representing the outputs of M geographically distributed sensors, as depicted in Fig. 1. The received signals are assumed to have been filtered to a band of interest and thus will generally be complex even if the sensor measures a real physical quantity (e.g., sound pressure). For $m = 1, \ldots, M$, let $\mathbf{x}_m \triangleq (x_{m,1}, \ldots, x_{m,N})^T$ be a complex random N vector obtained by sampling r_m. For convenience, compensation for propagation delays and doppler shifts are assumed to have been performed on the sensor outputs prior to the formation of the sample vectors $\mathbf{x}_m$.

The problem introduced heuristically above as detection of a common signal on M noisy channels is addressed in this paper by a detector designed to distinguish between hypotheses H_0, in which the vectors $\mathbf{x}_1, \ldots, \mathbf{x}_M$ contain independent samples of independent zero-mean Gaussian processes and its complement H_1. The variances of the processes in H_0 are not assumed to be identical or known. Hence, the hypotheses are invariant to multiplication of the sample vectors by arbitrary nonzero scalars (e.g., channel gains).

Since no signal model is assumed and the way in which the noise distorts the signal is not explicitly modeled, H_1 encompasses a broad class of scenarios. In particular, it includes the situation in which each sample vector contains a common nonzero deterministic signal component, possibly scaled by an arbitrary nonzero constant, in additive noise of the type just described. Receiver operating characteristic curves are developed by simulation of this particular H_1 scenario in a later section of this paper. It is important to note, however, that the approach will detect deviation from the H_0 assumptions whether or not it is due to the presence of a common signal on each channel.

III. The MSC Estimate

The role of magnitude-squared coherence in time-delay estimation and two-channel detection is well documented (see, for example, [5] and [6]). This section describes an alternative formulation of the MSC estimate that was introduced in [7] and [8] and shows how the H_0 distribution of the MSC estimate can be derived from this perspective. This treatment of the MSC estimate provides the groundwork for definition and analysis of the GC estimate in the next section.

Given two complex sequences represented as column vectors $\mathbf{x}_1$ and $\mathbf{x}_2$, the MSC estimate formed from them is given by [9]

$$\hat{\gamma}^2(\mathbf{x}_1, \mathbf{x}_2) \triangleq \frac{|\langle \mathbf{x}_1, \mathbf{x}_2 \rangle|^2}{||\mathbf{x}_1||^2 ||\mathbf{x}_2||^2} \tag{1}$$

where $\langle \mathbf{x}_1, \mathbf{x}_2 \rangle$ is the inner product of $\mathbf{x}_1$ and $\mathbf{x}_2$ defined by

$$\langle \mathbf{x}_1, \mathbf{x}_2 \rangle \triangleq \sum_{k=1}^{N} x_{1,k} x_{2,k}^*.$$

In these expressions, x^* denotes the complex conjugate of x, and $||\mathbf{x}||^2 \triangleq \langle \mathbf{x}, \mathbf{x} \rangle$ is the squared magnitude of $\mathbf{x}$.

A. Properties of the MSC Estimate

Assuming neither $\mathbf{x}_1$ nor $\mathbf{x}_2$ is identically zero, it is straightforward to show that the MSC estimate has the following properties:

1) $0 \leq \hat{\gamma}^2(\mathbf{x}_1, \mathbf{x}_2) \leq 1$.
2) $\hat{\gamma}^2(\mathbf{x}_1, \mathbf{x}_2) = 0$ if and only if $\mathbf{x}_1$ and $\mathbf{x}_2$ are orthogonal.
3) $\hat{\gamma}^2(\mathbf{x}_1, \mathbf{x}_2) = 1$ if and only if $\mathbf{x}_1$ and $\mathbf{x}_2$ are collinear (i.e., $\mathbf{x}_1 = \lambda \mathbf{x}_2$ for some nonzero complex scalar λ).
4) $\hat{\gamma}^2(\mathbf{x}_2, \mathbf{x}_1) = \hat{\gamma}^2(\mathbf{x}_1, \mathbf{x}_2)$ for all complex vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ (i.e., the order in which the channels are indexed does not matter).
5) $\hat{\gamma}^2(\lambda_1 \mathbf{x}_1, \lambda_2 \mathbf{x}_2) = \hat{\gamma}^2(\mathbf{x}_1, \mathbf{x}_2)$ for all nonzero scalars λ_1 and λ_2 (i.e., the estimate is invariant with respect to constant gain factors on either channel).

B. H_0 Distribution of the MSC Estimate

Knowledge of the statistical behavior of the MSC estimate when no common signal is present on the two channels allows threshold values corresponding to particular false alarm probabilities to be calculated, thereby establishing the utility of the MSC estimate as a detection statistic. The probability distribution function of the MSC estimate was derived in [3] under the H_0 assumptions that $\mathbf{x}_1$ and $\mathbf{x}_2$ are independent white Gaussian sequences. In [9], this same distribution function was shown to hold, regardless of the statistical behavior of $\mathbf{x}_1$, provided that $\mathbf{x}_2$ is a white Gaussian sequence that is statistically independent of $\mathbf{x}_1$. A geometric argument showing that this distribution remains valid under even weaker H_0 assumptions was given in [4].

For the purpose of generalization to $M > 2$ sequences, it is useful to consider an equivalent formulation of the MSC estimate and to derive its H_0 distribution directly from this formulation.

The MSC estimate may be written as

$$\hat{\gamma}^2(\mathbf{x}_1, \mathbf{x}_2) = 1 - \frac{g(\mathbf{x}_1, \mathbf{x}_2)}{||\mathbf{x}_1||^2 ||\mathbf{x}_2||^2}$$

where

$$g(\mathbf{x}_1, \mathbf{x}_2) \triangleq \det \begin{bmatrix} \langle \mathbf{x}_1, \mathbf{x}_1 \rangle & \langle \mathbf{x}_1, \mathbf{x}_2 \rangle \\ \langle \mathbf{x}_2, \mathbf{x}_1 \rangle & \langle \mathbf{x}_2, \mathbf{x}_2 \rangle \end{bmatrix}.$$

This matrix of inner products is called a *Gram matrix* [10], and its determinant is known as a *Gram determinant.* As shown in [10], the Gram determinant $g(\mathbf{x}_1, \mathbf{x}_2)$ satisfies

$$g(\mathbf{x}_1, \mathbf{x}_2) = g(\mathbf{x}_2, \mathbf{x}_1) \tag{2}$$

and

$$g(\mathbf{x}_1, \mathbf{x}_2 + \lambda \mathbf{x}_1) = g(\mathbf{x}_1, \mathbf{x}_2) \tag{3}$$

for any complex number λ. In addition, if $\mathbf{x}_1$ and $\mathbf{x}_2$ are orthogonal, the Gram matrix is diagonal, and $g(\mathbf{x}_1, \mathbf{x}_2) = ||\mathbf{x}_1||^2 ||\mathbf{x}_2||^2$.

Derivation of the H_0 distribution of the MSC estimate may be approached by a Gram–Schmidt procedure [11] on the Gram matrix. Suppose that $\mathbf{x}_1$ and $\mathbf{x}_2$ are arbitrary nonzero complex N vectors and define

$$\mathbf{y}_1 \triangleq \mathbf{x}_1$$

and

$$\mathbf{y}_2 \triangleq \mathbf{x}_2 - P(\mathbf{x}_2 | \mathbf{x}_1)$$

where $P(\mathbf{x}_2|\mathbf{x}_1)$ denotes the orthogonal projection of $\mathbf{x}_2$ onto the 1-D subspace of complex N-dimensional space $\mathbb{C}^N$ spanned by $\mathbf{x}_1$, i.e.

$$P(\mathbf{x}_2 | \mathbf{x}_1) \triangleq \left[\frac{\langle \mathbf{x}_2, \mathbf{x}_1 \rangle}{\langle \mathbf{x}_1, \mathbf{x}_1 \rangle} \right] \mathbf{x}_1.$$

Property (3) implies $g(\mathbf{y}_1, \mathbf{y}_2) = g(\mathbf{x}_1, \mathbf{x}_2)$. In addition, $\mathbf{y}_1$ and $\mathbf{y}_2$ are orthogonal so that $g(\mathbf{y}_1, \mathbf{y}_2) = ||\mathbf{y}_1||^2 ||\mathbf{y}_2||^2$. Hence, the MSC estimate may be written as

$$\hat{\gamma}^2(\mathbf{x}_1, \mathbf{x}_2) = 1 - z$$

where

$$z \triangleq \frac{g(\mathbf{x}_1, \mathbf{x}_2)}{||\mathbf{x}_1||^2 ||\mathbf{x}_2||^2} = \frac{g(\mathbf{y}_1, \mathbf{y}_2)}{||\mathbf{x}_1||^2 ||\mathbf{x}_2||^2} = \frac{||\mathbf{y}_2||^2}{||\mathbf{x}_2||^2}.$$

By definition of $\mathbf{y}_2$, $\mathbf{x}_2 = \mathbf{y}_2 + P(\mathbf{x}_2|\mathbf{x}_1)$. Because $\mathbf{y}_2$ and $P(\mathbf{x}_2|\mathbf{x}_1)$ are orthogonal, the Pythagorean theorem implies

$$||\mathbf{x}_2||^2 = ||\mathbf{y}_2||^2 + ||P(\mathbf{x}_2|\mathbf{x}_1)||^2$$

Hence

$$z = \frac{||\mathbf{x}_2||^2 - ||P(\mathbf{x}_2|\mathbf{x}_1)||^2}{||\mathbf{x}_2||^2}.$$

To obtain the H_0 distribution of the MSC estimate, the distribution of z under the assumption that $\mathbf{x}_1$ and $\mathbf{x}_2$ are independent complex white Gaussian noise sequences remains to be determined. Denote $\mathbf{x}_1 = (a_1 + ib_1, \ldots, a_N + ib_N)^T \in \mathbb{C}^N$, and define orthogonal vectors $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ in real $2N$-dimensional space $\mathbb{R}^{2N}$ by

$$\boldsymbol{\beta}_1 \triangleq (a_1, \ldots, a_N, b_1, \ldots, b_N)^T$$

and

$$\boldsymbol{\beta}_2 \triangleq (-b_1, \ldots, -b_N, a_1, \ldots, a_N)^T.$$

Note that $||\boldsymbol{\beta}_1|| = ||\boldsymbol{\beta}_2|| = ||\mathbf{x}_1||$ so that the vectors $\hat{\boldsymbol{\beta}}_1 \triangleq \boldsymbol{\beta}_1/||\boldsymbol{\beta}_1||$ and $\hat{\boldsymbol{\beta}}_2 \triangleq \boldsymbol{\beta}_2/||\boldsymbol{\beta}_1||$ are orthonormal in $\mathbb{R}^{2N}$. Thus, the set $\{\hat{\boldsymbol{\beta}}_1, \hat{\boldsymbol{\beta}}_2\}$ can be extended to an orthonormal basis $\{\hat{\boldsymbol{\beta}}_1, \hat{\boldsymbol{\beta}}_2, \hat{\boldsymbol{\beta}}_3, \ldots, \hat{\boldsymbol{\beta}}_{2N}\}$ of $\mathbb{R}^{2N}$ [11]. Writing

$$\mathbf{B} \triangleq \begin{bmatrix} | & & | \\ \hat{\boldsymbol{\beta}}_1 & \cdots & \hat{\boldsymbol{\beta}}_{2N} \\ | & & | \end{bmatrix}$$

an arbitrary vector $\boldsymbol{\alpha} \in \mathbb{R}^{2N}$ is expressed in terms of this new basis by $\tilde{\boldsymbol{\alpha}} = \mathbf{B}^T\boldsymbol{\alpha}$. Moreover, since $\mathbf{B}$ is a unitary matrix, inner products are invariant under the basis change, and in particular, $||\tilde{\boldsymbol{\alpha}}|| = ||\boldsymbol{\alpha}||$. Applying the basis change to $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ yields $\tilde{\boldsymbol{\beta}}_1 = (||\boldsymbol{\beta}_1||, 0, \ldots, 0)^T$ and $\tilde{\boldsymbol{\beta}}_2 = (0, ||\boldsymbol{\beta}_2||, 0, \ldots, 0)^T$.

Now, denote $\mathbf{x}_2 \triangleq (u_1 + iv_1, \ldots, u_N + iv_N)^T \in \mathbb{C}^N$, and define $\boldsymbol{\alpha} \triangleq (u_1, \ldots, u_N, v_1, \ldots, v_N)^T \in \mathbb{R}^{2N}$. Denoting $\tilde{\boldsymbol{\alpha}} = (w_1, \ldots, w_{2N})^T$

$$||\mathbf{x}_2||^2 = ||\boldsymbol{\alpha}||^2 = ||\tilde{\boldsymbol{\alpha}}||^2 = \sum_{n=1}^{2N} w_n^2.$$

Similarly

$$\begin{aligned} ||P(\mathbf{x}_2|\mathbf{x}_1)||^2 &= \frac{\langle \boldsymbol{\alpha}, \boldsymbol{\beta}_1 \rangle^2}{||\boldsymbol{\beta}_1||^2} + \frac{\langle \boldsymbol{\alpha}, \boldsymbol{\beta}_2 \rangle^2}{||\boldsymbol{\beta}_2||^2} \\ &= \frac{\langle \tilde{\boldsymbol{\alpha}}, \tilde{\boldsymbol{\beta}}_1 \rangle^2}{||\boldsymbol{\beta}_1||^2} + \frac{\langle \tilde{\boldsymbol{\alpha}}, \tilde{\boldsymbol{\beta}}_2 \rangle^2}{||\boldsymbol{\beta}_2||^2} = w_1^2 + w_2^2 \end{aligned}$$

Thus

$$z = \frac{\sum_{n=1}^{2N} w_n^2 - \sum_{n=1}^{2} w_n^2}{\sum_{n=1}^{2N} w_n^2} = \frac{\sum_{n=3}^{2N} w_n^2}{\sum_{n=1}^{2N} w_n^2}.$$

If $u_1, \ldots, u_N, v_1, \ldots, v_N$ are independent, zero-mean Gaussian random variables with common variance σ^2, then so are $w_1, \ldots, w_{2N}$. Hence, z has the beta distribution $\beta(2N-2, 2)$ of $2N-2$ and two degrees of freedom [12]. This implies that $\hat{\gamma}^2(\mathbf{x}_1, \mathbf{x}_2) = 1 - z$ has the $\beta(2, 2N-2)$ distribution, i.e.

$$\Pr\{\hat{\gamma}^2(\mathbf{x}_1, \mathbf{x}_2) \le r\} = 1 - (1-r)^{N-1} \quad 0 \le r \le 1. \tag{4}$$

This, of course, is the same H_0 distribution obtained for the MSC estimate in [3] and [4].

IV. The GC Estimate

Viewing the MSC estimate in terms of the determinant of a 2×2 Gram matrix suggests a direct generalization to M channels in terms of the determinant of a $M \times M$ Gram matrix. Given M complex sequences represented by vectors $\mathbf{x}_1, \ldots, \mathbf{x}_M$, the *generalized coherence* (GC) estimate $\hat{\gamma}_M^2(\mathbf{x}_1, \ldots, \mathbf{x}_M)$ is defined by

$$\hat{\gamma}_M^2(\mathbf{x}_1, \ldots, \mathbf{x}_M) \triangleq 1 - \frac{g(\mathbf{x}_1, \ldots, \mathbf{x}_M)}{||\mathbf{x}_1||^2 \cdots ||\mathbf{x}_M||^2}$$

where $g(\mathbf{x}_1, \ldots, \mathbf{x}_M)$ is the determinant of an $M \times M$ Gram matrix whose (i,j)th element is the inner product $\langle \mathbf{x}_i, \mathbf{x}_j \rangle$. From above, the two-channel GC estimate $\hat{\gamma}_2^2(\mathbf{x}_1, \mathbf{x}_2)$ is identical to the MSC estimate $\hat{\gamma}^2(\mathbf{x}_1, \mathbf{x}_2)$.

A. Properties of the GC Estimate

Assuming no $\mathbf{x}_m$ for $m = 1, \ldots, M$ is identically zero, properties of the Gram determinant $g(\mathbf{x}_1, \ldots, \mathbf{x}_M)$ given in [10] imply the following properties of the GC estimate:

1) $0 \le \hat{\gamma}_M^2(\mathbf{x}_1, \ldots, \mathbf{x}_M) \le 1$.
2) $\hat{\gamma}_M^2(\mathbf{x}_1, \ldots, \mathbf{x}_M) = 0$ if and only if $\mathbf{x}_j$ and $\mathbf{x}_k$ are orthogonal for all $j = 1, \ldots, M$ and $k = 1, \ldots, M$ with $j \neq k$.
3) $\hat{\gamma}_M^2(\mathbf{x}_1, \ldots, \mathbf{x}_M) = 1$ if and only if $\{\mathbf{x}_1, \ldots, \mathbf{x}_M\}$ is a linearly dependent set of vectors in $\mathbb{C}^N$.
4) $\hat{\gamma}_M^2(\mathbf{x}_1, \ldots, \mathbf{x}_j, \ldots, \mathbf{x}_k, \ldots, \mathbf{x}_M) = \hat{\gamma}_M^2(\mathbf{x}_1, \ldots, \mathbf{x}_k, \ldots, \mathbf{x}_j, \ldots, \mathbf{x}_M)$ for all j and k with $1 \le j < k \le M$ (i.e., the order in which the channels are indexed does not matter).
5) $\hat{\gamma}_M^2(\lambda_1\mathbf{x}_1, \ldots, \lambda_M\mathbf{x}_M) = \hat{\gamma}_M^2(\mathbf{x}_1, \ldots, \mathbf{x}_M)$ for all nonzero scalars $\lambda_1, \ldots, \lambda_M$ (i.e., the estimate is invariant with respect to constant gain factors on the channels).

Note the similarity of these properties and those of the MSC estimate listed above. In particular, property 4) implies that no channel is specially distinguished from the others to serve as a "reference channel," as is the case with the multiple coherence (MC) estimate—a statistic employed in pioneering work on multiple-channel detection [13]. Additional comparisons between the properties and detection performances of the MC and GC estimates are presented in Section VII.

B. H_0 Distribution of the GC Estimate

The Gram–Schmidt procedure used above to derive the H_0 distribution of the MSC estimate may be extended to determine the H_0 distribution of the GC estimate. Given M complex Nvectors $\mathbf{x}_1, \ldots, \mathbf{x}_M$, define

$$\mathbf{y}_1 \triangleq \mathbf{x}_1$$

and, for $m = 2, \ldots, M$, define

$$\mathbf{y}_m \triangleq \mathbf{x}_m - P(\mathbf{x}_m | \mathbf{x}_1, \ldots, \mathbf{x}_{m-1})$$

where $P(\mathbf{x}_m | \mathbf{x}_1, \ldots, \mathbf{x}_{m-1})$ denotes the orthogonal projection of $\mathbf{x}_m$ onto the linear subspace of $\mathbb{C}^N$ spanned by $\{\mathbf{x}_1, \ldots, \mathbf{x}_{m-1}\}$. As shown in [10], $g(\mathbf{x}_1, \ldots, \mathbf{x}_k + \lambda \mathbf{x}_j, \ldots, \mathbf{x}_M) = g(\mathbf{x}_1, \ldots, \mathbf{x}_M)$ for any complex number λ and $j \neq k$. Thus

$$z \triangleq \frac{g(\mathbf{x}_1, \ldots, \mathbf{x}_M)}{\|\mathbf{x}_1\|^2 \cdots \|\mathbf{x}_M\|^2} = \frac{g(\mathbf{y}_1, \ldots, \mathbf{y}_M)}{\|\mathbf{x}_1\|^2 \cdots \|\mathbf{x}_M\|^2}. \tag{5}$$

Moreover, since $\mathbf{y}_j$ and $\mathbf{y}_k$ are orthogonal for $j \neq k$, $g(\mathbf{y}_1, \ldots, \mathbf{y}_M) = \|\mathbf{y}_1\|^2 \cdots \|\mathbf{y}_M\|^2$. Therefore, z can be expressed as the product of $M - 1$ terms, i.e.

$$z = z_2 \cdots z_M$$

with

$$z_m \triangleq \frac{\|\mathbf{y}_m\|^2}{\|\mathbf{x}_m\|^2}$$

for $m = 2, \ldots, M$.

Suppose now that the real and imaginary parts of each component of $\mathbf{x}_2$ are independent zero-mean Gaussian random variables with common variance σ^2. The argument given in the two-channel case implies

$$z_2 = \frac{\sum_{n=3}^{2N} w_n^2}{\sum_{n=1}^{2N} w_n^2}$$

where $w_1, \ldots, w_{2N}$ are also independent, zero-mean Gaussian random variables with common variance σ^2. Assuming the sequences $\mathbf{x}_m$ are independent and identically distributed for $m = 2, \ldots, M$, a direct extension of the argument given in the two-channel case shows

$$\|\mathbf{y}_m\|^2 = \|\mathbf{x}_m\|^2 - \|P(\mathbf{x}_m | \mathbf{x}_1, \ldots, \mathbf{x}_{m-1})\|^2$$

so that

$$z_m = \frac{\|\mathbf{y}_m\|^2}{\|\mathbf{x}_m\|^2} = \frac{\|\mathbf{x}_m\|^2 - \|P(\mathbf{x}_m | \mathbf{x}_1, \ldots, \mathbf{x}_{m-1})\|^2}{\|\mathbf{x}_m\|^2} = \frac{\sum_{n=2m-1}^{2N} w_n^2}{\sum_{n=1}^{2N} w_n^2}$$

where $w_1, \ldots, w_{2N}$ are also independent zero-mean Gaussian random variables with common variance σ^2. Thus, z_m has a beta distribution of $2N - 2(m-1)$ and $2(m-1)$ degrees of freedom, defined explicitly in [12] by

$$\Pr\{z_m \leq r\} = 1 - (1-r)^{N-1} \sum_{j=0}^{N-m} \binom{N-1}{j} \left[\frac{r}{1-r}\right]^j \tag{6}$$

for $2 \leq m \leq M$ and $0 \leq r \leq 1$.

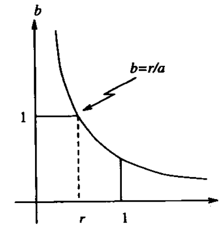


Fig. 2. Distribution of the GC estimate for three channels is obtained by integrating the joint density function of z_2 and z_3 over the planar region pictured.

Although the distributions of the independent factors z_m have a relatively simple polynomial structure, it is still a rather involved task to obtain a closed-form equation for the distribution of $\hat{\gamma}_M^2(\mathbf{x}_1, \ldots, \mathbf{x}_M) = 1 - z_2 \cdots z_M$ from the distributions of the z_m. Equation (6) yields the recursion formula

$$\begin{cases} \Pr\{z_2 \leq r\} = r^{N-1} \\ \Pr\{z_{m+1} \leq r\} = \Pr\{z_m \leq r\} + \binom{N-1}{N-m} r^{N-m}(1-r)^{m-1} \end{cases} \tag{7}$$

for fixed N and $2 \leq m \leq N$. This formula may be applied directly to determine the H_0 distribution function of the GC estimate for small numbers of channels. It also leads to another recursion formula that describes how the H_0 distribution function of $\hat{\gamma}_{M+1}^2$ can be obtained from that of $\hat{\gamma}_M^2$. This formula will be developed in a later section.

C. Distribution of the Three-Channel GC Estimate

To calculate the distribution function of $\hat{\gamma}_3^2 = 1 - z_2 z_3$, note that the recursion formula (7) provides

$$\begin{aligned} \Pr\{z_3 \leq r\} &= r^{N-1} + (N-1) r^{N-2}(1-r) \\ &= (N-1) r^{N-2} - (N-2) r^{N-1} \end{aligned}$$

Hence, the joint density function $f_{z_2 z_3}$ of the independent random variables z_2 and z_3 is

$$f_{z_2 z_3}(a, b) = (N-1)^2 (N-2) a^{N-2} [b^{N-3} - b^{N-2}]$$

and $\Pr\{z_2 z_3 \leq r\}$ is given by the integral of $f_{z_2 z_3}$ over the subset of the unit square in the $a - b$ plane on which $ab \leq r$ (Fig. 2). Thus, for $0 \leq r \leq 1$,

$$\begin{aligned} \Pr\{z_2 z_3 \leq r\} &= \int_0^r \int_0^1 f_{z_2 z_3}(a, b)\, db\, da \\ &\quad + \int_r^1 \int_0^{r/a} f_{z_2 z_3}(a, b)\, db\, da \\ &= r^{N-1} + (N-1)(N-2) r^{N-1} \log(r) \\ &\quad + (N-1)^2 [r^{N-2} - r^{N-1}] \end{aligned}$$

so that

$$\begin{aligned} \Pr\{\hat{\gamma}_3^2 \leq r\} &= 1 - \Pr\{z_2 z_3 \leq 1 - r\} \\ &= 1 - (1-r)^{N-1} \\ &\quad - (N-1)(N-2)(1-r)^{N-1} \log(1-r) \\ &\quad - (N-1)^2 [(1-r)^{N-2} - (1-r)^{N-1}]. \end{aligned}$$

D. Recursion Formula for the Distribution of the Multichannel GC Estimate

Having derived the H_0 distribution functions of the GC estimate using $M = 2$ and $M = 3$ channels explicitly, a recursion formula for the distribution of the GC estimate using larger numbers of channels may now be obtained as follows. Note that the independence of the random variables $\zeta_m \triangleq z_2 z_3 \cdots z_m$ and z_{m+1} implies that

$$\begin{aligned}\Pr\{z_2 \cdots z_m z_{m+1} \le r\} &= \int_0^r \int_0^1 f_{\zeta_m}(a) f_{z_{m+1}}(b)\, db\, da \\ &\quad + \int_r^1 \int_0^{r/a} f_{\zeta_m}(a) f_{z_{m+1}}(b)\, db\, da \\ &= \Pr\{\zeta_m \le r\} + \int_r^1 f_{\zeta_m}(a) \Pr\{z_{m+1} \le r/a\}\, da.\end{aligned}$$

Using the recursion formula for the distribution of z_{m+1} from (7), this expression yields

$$\begin{aligned}\Pr\{\zeta_{m+1} \le r\} &= \Pr\{\zeta_m \le r\} + \int_r^1 f_{\zeta_m}(a) \Pr\{z_m \le r/a\}\, da \\ &\quad + \binom{N-1}{N-m} \int_r^1 f_{\zeta_m}(a) \left(\frac{r}{a}\right)^{N-m} \left(1 - \frac{r}{a}\right)^{m-1} da.\end{aligned}$$

Since $\Pr\{\hat{\gamma}_{m+1}^2 \le r\} = 1 - \Pr\{\zeta_{m+1} \le 1 - r\}$, this expression describes how the H_0 distribution function of $\hat{\gamma}_{m+1}^2$ can be obtained from that of $\hat{\gamma}_m^2$.

V. Detection Thresholds

In the context of multiple-channel signal detection, the principal value of knowing the H_0 distribution function of the GC estimate is its utility in determining detection thresholds corresponding to desired false alarm probabilities. Such thresholds provide the basis for deciding whether to hypothesize the presence or absence of a common signal on the noisy channels given a GC value computed using actual data sequences from the channels. Tables I and II give values of the GC estimate corresponding to a range of probabilities of false alarm P_f and sample sequence lengths N for two and three channels, respectively.

VI. Detection Performance

This section discusses the performance of GC detectors in detecting the presence of a common white Gaussian signal in independent channels of additive white Gaussian noise. The distribution of the GC estimate with signal absent is known from the above derivation, but its distribution under this particular set of H_1 assumptions is not known and must be evaluated using computer simulations. Although the performance of detectors based on the MSC have already been documented, selected results for a two-channel GC detector are included here for reference in subsequent discussions of detector performance. All simulations used independent sequences of white Gaussian noise and a white Gaussian signal that was independent from each of the noise sequences.

TABLE I

Values of $\hat{\gamma}_2^2$ Corresponding to Certain Probabilities of False Alarm

P_f	$N = 4$	$N = 8$	$N = 16$	$N = 32$	$N = 64$	$N = 128$	$N = 256$
10^{-1}	.5358411	.2803143	.1423041	.0715855	.0358891	.0179672	.0089891
10^{-2}	.7845565	.4820525	.2643577	.1380464	.0704902	.0356116	.0178974
10^{-3}	.9000000	.6272406	.3690427	.1997498	.1038495	.0529390	.0267256
10^{-4}	.9535841	.7317304	.4588305	.2570360	.1360116	.0699551	.0354745
10^{-5}	.9784557	.8069302	.5358411	.3102215	.1670193	.0866654	.0441447
10^{-6}	.9900000	.8610505	.6018928	.3595996	.1969143	.1030755	.0527370

TABLE II

Values of $\hat{\gamma}_3^2$ Corresponding to Certain Probabilities of False Alarm

P_f	$N = 4$	$N = 8$	$N = 16$	$N = 32$	$N = 64$	$N = 128$	$N = 256$
10^{-1}	.8754568	.5521470	.3046456	.1593690	.0814290	.0411485	.0206850
10^{-2}	.9646636	.7195777	.4367786	.2398269	.1255404	.0642102	.0324684
10^{-3}	.9892299	.8176667	.5356765	.3067238	.1640644	.0848355	.0431347
10^{-4}	.9966401	.8793963	.6140143	.3651875	.1993141	.1041324	.0532235
10^{-5}	.9989429	.9194506	.6775521	.4173248	.2321731	.1225105	.0629335
10^{-6}	.9996663	.9458743	.7297485	.4643189	.2631002	.1401745	.0723633

TABLE III

Equichannel SNR Values (in Decibels) Necessary to Acheive a 50% Detection Probability Using a Two-Channel GC Detector (from Simulations)

P_f	$N = 4$	8	16	32	64	128	256
10^{-1}	1.2	−0.3	−1.5	−2.6	−3.6	−4.3	−5.2
10^{-2}	3.6	1.4	−0.1	−1.4	−2.5	−3.2	−4.1
10^{-3}	5.5	2.5	0.8	−0.7	−1.8	−2.7	−3.6
10^{-4}	7.3	3.5	1.4	−0.1	−1.4	−2.2	−3.2
10^{-5}	9.0	4.4	2.0	0.3	−0.9	−1.9	−2.9
10^{-6}	10.7	5.2	2.5	0.7	−0.6	−1.6	−2.6

TABLE IV

Equichannel SNR Values (in Decibels) Necessary to Acheive a 50% Detection Probability Using a Three-Channel GC Detector (from Simulations)

P_f	$N = 4$	8	16	32	64	128	256
10^{-1}	1.0	−0.8	−2.2	−3.5	−4.1	−4.9	−6.1
10^{-2}	3.0	0.5	−1.0	−2.1	−3.2	−4.1	−5.0
10^{-3}	4.6	1.4	−0.3	−1.5	−2.7	−3.6	−4.3
10^{-4}	6.0	2.1	0.3	−1.1	−2.3	−3.2	−4.2
10^{-5}	7.3	2.8	0.7	−0.8	−2.0	−2.9	−3.9
10^{-6}	8.6	3.4	1.1	−0.5	−1.7	−2.8	−3.5

Table III shows performance results for a two-channel GC detector (MSC detector) assuming the signal-to-noise ratio (SNR) is identical on the two channels. The SNR necessary to achieve a detection probability of 50% is shown for a range of false alarm probabilities (P_f) and sequence lengths (N). Fig. 3 contains receiver operating characteristic (ROC) curves for two-channel GC detectors with $N = 128$ and $N = 256$, also assuming equal SNR's on the two channels. Corresponding performance data for three-channel GC detectors with equal SNR's on all channels are given in Table IV and Fig. 4.

Fig. 5 depicts performance curves for two- and three-channel GC detectors where the SNR's on the channels are unequal. All of the curves in this figure correspond to a probability of detection (P_d) of 0.5 and a probability of false alarm (P_f) of 10^{-3} for GC detectors using sequences of length $N = 64$. The dashed curve is the locus of $(\mathrm{SNR}_1, \mathrm{SNR}_2)$ pairs necessary for a two-channel GC detector (MSC detector) to achieve this performance level. Performance curves of this type are well established for MSC detectors. For fixed N, they show a trend of decreasing SNR_2 required to achieve the desired P_d and P_f levels as SNR_1 increases. The curves are approximately hyperbolic in shape and asymptotically approach a minimum value of SNR_2 as SNR_1 approaches infinity, which represents the minimum detectable SNR on channel 2 when channel 1 contains an exact signal replica (i.e., a "matched filter").

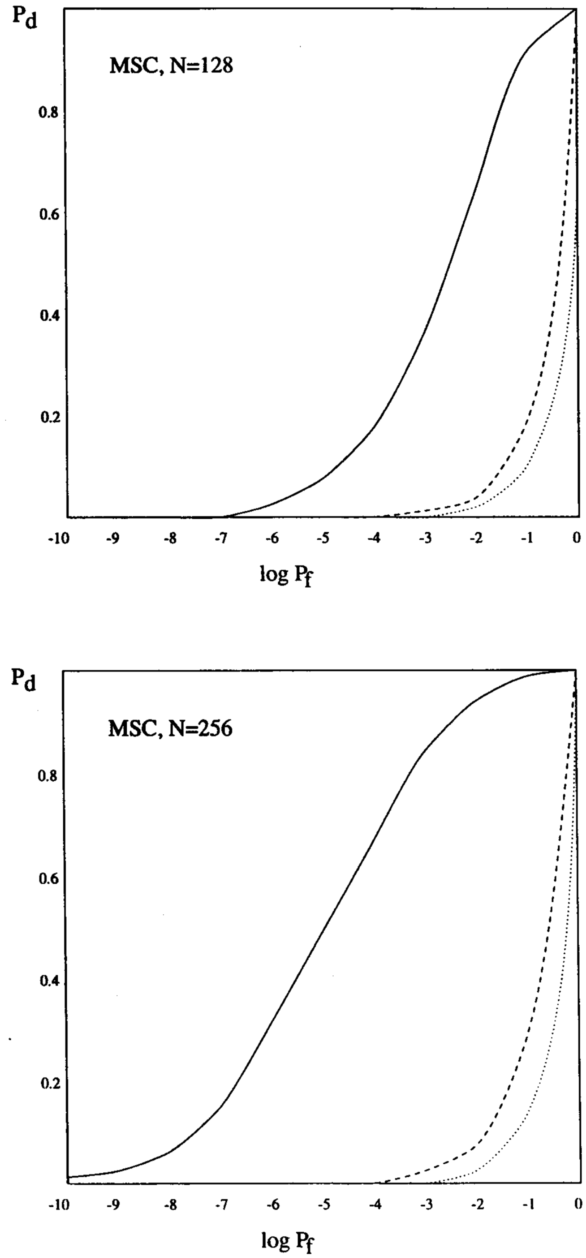


Fig. 3. Detection probability (P_d) versus false alarm probability (P_f) for a MSC detector with sequence lengths $N = 128$ (top) and $N = 256$ (bottom). SNR's are equal on both channels. Solid lines are for -3 dB, dashed lines for -6 dB, and dotted lines for -9 dB. Note that the MSC detector is the same as a two-channel GC detector.

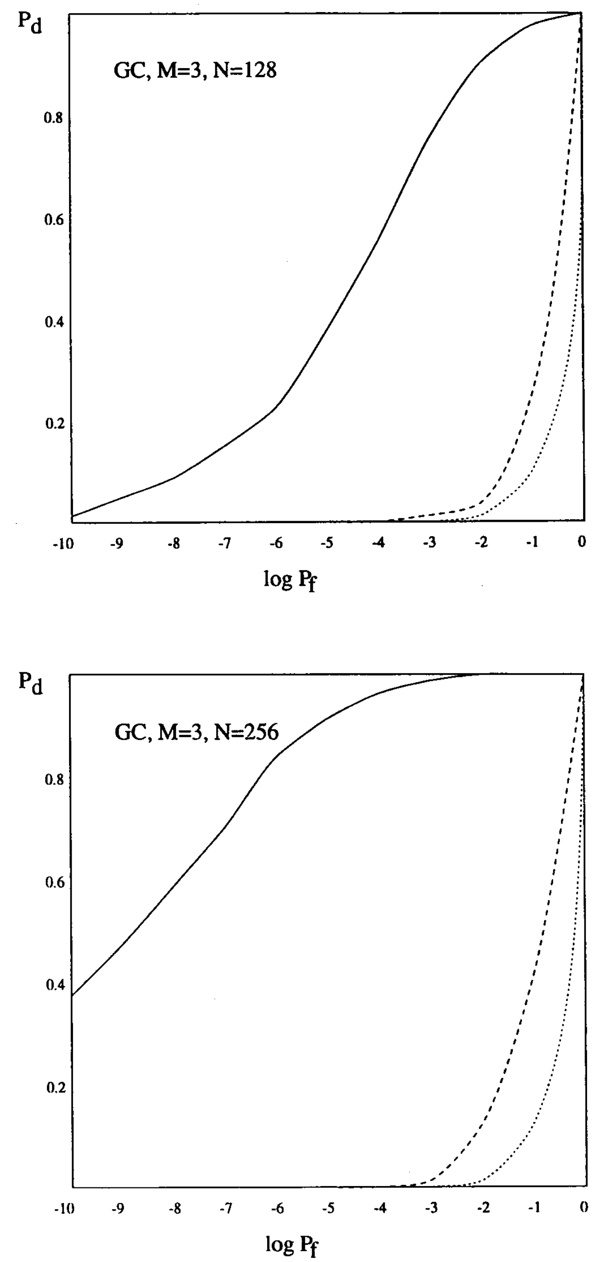


Fig. 4. Detection probability (P_d) versus false alarm probability (P_f) for a three-channel GC detector with sequence lengths $N = 128$ (top) and $N = 256$ (bottom). SNR's are equal on all three channels. Solid lines are for -3 dB, dashed lines for -6 dB, and dotted lines for -9 dB.

Each of the solid curves in Fig. 5 corresponds to a fixed SNR on channel 3 of a three-channel GC detector. Given that value of SNR_3, the curves are loci of $(\mathrm{SNR}_1, \mathrm{SNR}_2)$ pairs that are needed to achieve the desired P_d and P_f. Note that the GC estimate treats all channels equally; therefore, the curves would not change if the indexing of the channels were altered.

The shapes of these curves indicate an important characteristic of multiple-channel GC detectors: A sufficiently strong signal on any two channels will cause the GC estimate to exceed the detection threshold. In the case represented in Fig. 5, the detector will detect with $\mathrm{SNR}_3 = 0$ dB and $\mathrm{SNR}_2 = -2$ dB regardless of the SNR on channel 1. The reason for this is evident from the definition of the GC estimate: If $\mathbf{x}_i = \mathbf{x}_j$ for any $i \neq j$, the Gram matrix will be singular, and the GC estimate will be one. Some implications of this property will be discussed later in this paper.

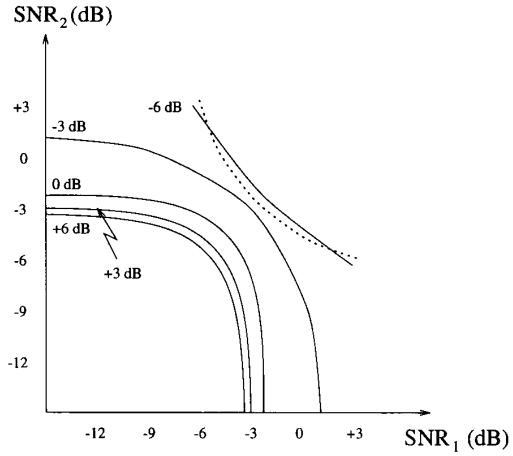


Fig. 5. Detection curves for two-channel (dashed) and three-channel (solid) GC detectors. The two-channel curve shows the combinations of channel 1 SNR and channel 2 SNR needed to achieve detection performance ($P_d = 0.5$, $P_f = 10^{-3}$) with $N = 64$. Each three-channel curve indicates the combinations of channel 1 SNR and channel 2 SNR needed to achieve this same performance level assuming a given SNR on channel 3.

For the situation depicted in Fig. 5, a fixed SNR of -6 dB on channel 3 causes the three-channel GC detector to behave approximately like a two-channel GC detector as a function of the SNR's on channels 1 and 2. More generally, there will be some minimum SNR on channel $M+1$ that will enable a $M+1$-channel GC detector using channels $1, \ldots, M+1$ to perform better than a M-channel GC detector using channels $1, \ldots, M$. On the other hand, there are cases in which a three-channel detector will detect when a two-channel detection is not possible using any pair of the three channels. With $N = 64$ and $P_f = 10^{-3}$, for example, a detection probability of 0.5 is attained by a three-channel GC detector when $\mathrm{SNR}_1 = \mathrm{SNR}_2 = \mathrm{SNR}_3 = -2.7$ dB (Table IV). This SNR on both channels is not sufficient to yield $P_d = 0.5$ for a two-channel GC detector with the same N and P_f (Table III).

VII. Comparison with Multiple Coherence

The multiple coherence of M random processes is defined in numerous references (e.g., [14], [15]), and an estimator of multiple coherence called the "sample multiple coherence" [16], or simply the "MC estimate" [13], is reasonably well known. The optimality of MC in determining a linear time-invariant relationship among M processes under specific assumptions is discussed in [14]. An approach for detecting the presence of a common signal M noisy channels based on the MC estimate was formulated in [13]. This MC detector is similar to the GC detector discussed above in that it does not explicitly rely on *a priori* knowledge about the structure of the signal to be detected. Moreover, like the GC estimate, the MC estimate reduces to the MSC estimate for $M = 2$.

This section compares the performance of GC and MC detectors in detecting a white Gaussian signal in independent channels of additive white Gaussian noise. Note that the optimality of MC mentioned above does not apply in this situation because the relationship to be detected among the M sensor output processes is not linear. Nevertheless, the authors are aware that the MC approach has been used effectively in applications involving additive noise.

Since the GC and MC estimates both reduce to the MSC estimate when only two data sequences are involved, the simplest case in which comparison is meaningful involves three channels. Thus, the three-channel case is considered here.

A. Three-Channel MC Estimate

When more than two channels are involved, the MC estimate requires that one of the channels be distinguished as a "reference channel." For the purposes of this paper, the channels are statistically indistinguishable, and the first channel will always be the reference. With this understanding, the three-channel MC estimate is obtained from complex sequences $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ of length N by

$$\hat{\gamma}^2_{\mathrm{MC}}(\mathbf{x}_1 : \mathbf{x}_2, \mathbf{x}_3) = \frac{\|\mathbf{x}_3\|^2 |\langle \mathbf{x}_1, \mathbf{x}_2 \rangle|^2 + \|\mathbf{x}_2\|^2 |\langle \mathbf{x}_1, \mathbf{x}_3 \rangle|^2}{\|\mathbf{x}_1\|^2 \|\mathbf{x}_2\|^2 \|\mathbf{x}_3\|^2 - \|\mathbf{x}_1\|^2 |\langle \mathbf{x}_2, \mathbf{x}_3 \rangle|^2} - \frac{2\mathrm{Re}(\langle \mathbf{x}_1, \mathbf{x}_2 \rangle \langle \mathbf{x}_2, \mathbf{x}_3 \rangle \langle \mathbf{x}_3, \mathbf{x}_1 \rangle)}{\|\mathbf{x}_1\|^2 \|\mathbf{x}_2\|^2 \|\mathbf{x}_3\|^2 - \|\mathbf{x}_1\|^2 |\langle \mathbf{x}_2, \mathbf{x}_3 \rangle|^2}.$$

The H_0 distribution function of the three-channel MC estimate is given in [13] by

$$\Pr\{\hat{\gamma}^2_{\mathrm{MC}} \le r\} = 1 - (N-1)(1-r)^{N-2} + (N-2)(1-r)^{N-1}$$

for $0 \le r \le 1$.

B. Performance Comparison

Simulations corresponding to those used for measuring the performance of the three-channel GC detector were performed for a three-channel MC detector. Probabilities of detection estimated from simulations are plotted against theoretically predicted probabilities of false alarm in Fig. 6 for both three-channel MC and GC detectors with $N = 256$ and SNR values of -3 and -6 dB.

The trend seen in these figures is the same as that observed in several sets of simulation results: The performance of the three-channel GC detector is better than or equal to that of the three-channel MC detector. Of course, these results apply only to the three-channel setting with white Gaussian signal in additive white Gaussian noise. Performance comparisons have not been done for larger numbers of channels, for signals and noise having different statistical properties, or for situations in which the SNR's are different on the various channels.

VIII. Discussion

Before concluding the paper, a few comments regarding the GC estimate and its use as a multiple-channel detection statistic are presented in this section.

A. Geometrical Interpretation of the GC Estimate

As suggested by its formulation in terms of a matrix determinant, the GC estimate has an intuitively appealing geometrical interpretation. Heuristically, the Gram matrix determinant $g(\mathbf{x}_1, \ldots, \mathbf{x}_M)$ may be regarded as the squared

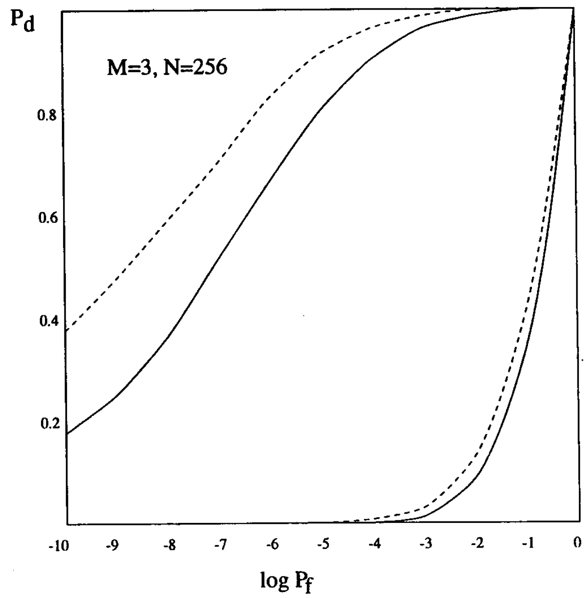


Fig. 6. Detection probability (P_d) versus false alarm probability (P_f) for three-channel MC (solid) and GC (dashed) detectors with sequence length $N = 256$. SNR's are equal on all three channels. The upper pair of curves is at -3 dB and the lower pair at -6 dB.

volume of a parallelepiped in complex N-dimensional space formed by the sample vectors $\mathbf{x}_1, \ldots, \mathbf{x}_M$. Normalizing by the product of the squared lengths of these vectors results in a number between zero and one that is subtracted from unity to give the GC estimate. This perspective is appealing because it makes several of the important properties of the GC estimate intuitively apparent. In particular, the volume in question clearly does not depend on the order in which the vectors are indexed. It will be zero (yielding $\hat{\gamma}_M^2 = 1$) if any two vectors are collinear because the dimensionality of the parallelepiped will be smaller than M. It will be one (yielding $\hat{\gamma}_M^2 = 0$) if and only if the vectors are an orthogonal set.

This perspective also helps to illustrate the fundamental difference between the GC and MC estimates. The MC estimate (again heuristically) measures the maximum coherence of the reference vector $\mathbf{x}_1$ with a linear combination of the remaining vectors $\mathbf{x}_2, \ldots, \mathbf{x}_M$. The dependence of the MC estimate on the choice of reference vector is thus clear.

B. Relationship to Wishart Distributed Matrices

With the H_0 assumptions described above, the Gram matrix belongs to the class of *complex Wishart distributed matrices*. The joint distribution of the elements of such matrices is described by Goodman in [16], who also points out their relationship to multiple coherence. An alternative derivation of the H_0 distribution of the GC estimate can be derived using Goodman's results. In particular, he shows that Wishart distributed matrices can be factored uniquely as $T^H T$, where T is an upper triangular matrix that is real on the main diagonal, and T^H denotes the conjugate transpose of T. Consequently, under the H_0 assumptions, the Gram matrix factors in this way, and its determinant may thus be expressed as

$$g(\mathbf{x}_1, \ldots, \mathbf{x}_M) = T_{1,1}^2 \cdots T_{m,m}^2.$$

It is not difficult to show that

$$||\mathbf{x}_m||^2 = \sum_{j=1}^{m} |T_{j,m}|^2$$

so that z defined in (5) factors as

$$z = \frac{T_{2,2}^2}{|T_{1,2}|^2 + T_{2,2}^2} \cdots \frac{T_{M,M}^2}{|T_{1,M}|^2 + \cdots T_{M,M}^2}$$

The remainder of the argument is essentially the same as the one given previously.

C. Statistical Foundation

Like the MC estimate, the GC estimate as defined above can be regarded as an estimator of an underlying statistical entity. In the GC case, this entity is a normalized version of the *generalized variance* of the processes $r_1, \ldots, r_M$ [18]. The authors are not aware of any results concerning the performance of the GC estimate in this context (i.e., its bias, consistency, or variance).

D. Issues in Practical Applications

Throughout this paper, it has been assumed that the GC approach is used to test for the presence of a common signal component in data that have already been appropriately adjusted to account for propagation delays and doppler shifts. In practice, the position and motion of the signal source is generally not known exactly, and many coherence estimates may need to be computed to search over a range of time delay and doppler shift combinations. Unfortunately, direct computation of the GC estimate is computationally intensive even for a small number of channels [17]. A special-purpose VLSI architecture for time-domain computation of GC estimates has been proposed by Lloyd [17]. Frequency-domain methods similar to the approach described in [3] have not been explored for the GC case.

It is apparent from the simulation results presented above (cf. Fig. 5) that the presence of a signal at sufficiently high SNR on any two channels is sufficient to cause the GC estimate to exceed the detection threshold. This may be a feature or a drawback, depending on the application. If, for example, a "three-channel detection" is obtained when the signal is strong on two channels but absent on the third, the resulting channels 1–3 and channels 2–3 time delay estimates should not be used in estimating the source location. In such situations (i.e., where only a true M-channel detection is desired), it is possible to discount detections resulting from the presence of the signal on fewer than M channels by considering lower order GC estimates in conjunction with the M-channel one or by examining the eigenvalues of the Gram matrix.

IX. Conclusion

In the simulation results presented above, though limited, the GC estimate shows promise as a multiple-channel detection statistic. All simulations done to date have indicated that the GC-based three-channel detector performs as well or better than the corresponding MC-based detector for signals in additive white Gaussian noise.

Some open problems related to GC-based detection include measurement of detector performance with more channels and different types of signal distortions (e.g., multiplicative noise). Problems involving analytical determination of the distribution of the GC estimate under particular sets of H_1 assumptions are also open.

The MSC estimate has been shown to be invariant with respect to the statistical behavior of $\mathbf{x}_1$, provided that $\mathbf{x}_2$ has a spherically symmetric distribution and is independent of $\mathbf{x}_1$ [4]. In particular, the detection thresholds remain valid if $\mathbf{x}_1$ is an exact signal replica under these assumptions on $\mathbf{x}_2$. Thus, the MSC estimate may be of value in active radar or sonar detection applications. A similar invariance result for the GC estimate has been conjectured and is supported by simulation results but has yet to be proven mathematically. Such a result would provide a basis for "multiple-channel matched filtering" in multistatic active detection scenarios.

Acknowledgments

The authors wish to thank the Associate Editor, Dr. I. M. G. Lourtie, and the reviewers for contributions that have significantly improved this paper. They are also grateful to Dr. G. C. Carter for his comments during the period this work was in progress.

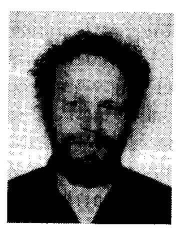
References

[1] M. E. Wang and M. J. Crocker, "On the application of coherence techniques for source identification in a multiple noise source environment," *J. Acoust. Soc. Amer.*, vol. 74, pp. 861–872, Sept. 1983.
[2] A. M. Zoubir and J. F. Böhme, "Bootstrap multiple tests applied to sensor location," *IEEE Trans. Signal Processing*, vol. SP-43, June 1995.
[3] G. C. Carter, C. Knapp, and A. H. Nuttall, "Estimation of the magnitude-squared coherence function via overlapped fast Fourier transform processing," *IEEE Trans. Audio Electroacoust.*, vol. AU-21, pp. 337–344, Aug. 1973.
[4] H. Gish and D. Cochran, "Invariance of the magnitude-squared coherence estimate with respect to second-channel statistics," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-35, pp. 1774–1776, Dec. 1987.
[5] G. C. Carter, "Coherence and time delay estimation," *Proc. IEEE*, vol. 75, pp. 236–255, Feb. 1987.
[6] ——, *Coherence and Time Delay Estimation*. New York: IEEE, 1993.
[7] H. Gish and D. Cochran, "Generalized coherence," in *Proc. IEEE Int. Conf. Acoust. Speech, Signal Processing*, vol. 5, 1987, pp. 2745–2748.
[8] D. Cochran and H. Gish, "Multiple-channel detection using generalized coherence," in *Proc. IEEE Int. Conf. Acoust. Speech, Signal Processing*, vol. 5, 1989, pp. 2883–2886.
[9] A. H. Nuttall, "Invariance of distribution of coherence estimate to second-channel statistics," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-29, pp. 120–122, Feb. 1981.
[10] P. J. Davis, *Interpolation and Approximation*. New York: Dover, 1975.
[11] P. R. Halmos, *Finite Dimensional Vector Spaces*. New York: Van Nostrand, 1958.
[12] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions*. New York: Dover, 1966.
[13] R. D. Trueblood and D. L. Alspach, "Multiple coherence as a detection statistic," NOSC Tech. Rep. 265, Naval Ocean Syst. Cent., 1978.
[14] D. R. Brillinger, *Time Series: Data Analysis and Theory*. San Francisco: Holden-Day, 1981.
[15] R. A. Johnson and D. W. Wichern, *Applied Multivariate Statistical Analysis*. Englewood Cliffs, NJ: Prentice-Hall, 1992.
[16] N. R. Goodman, "Statistical analysis based on a certain multivariate complex Gaussian distribution (An introduction)," *Ann. Math. Stat.*, vol. 34, no. 1, pp. 152–177, Mar. 1963.
[17] S. Lloyd, "Issues in multiple-channel coherence estimation," M.S. Thesis, Arizona State Univ., July 1992.
[18] S. S. Wilks, *Mathematical Statistics*. New York: Wiley, 1962.

Douglas Cochran (M'92) received the S.B. degree from the Massachusetts Institute of Technology in 1979 and the M.A. degree from the University of California in 1980, both in mathematics. He received the S.M. and Ph.D. degrees in applied mathematics from Harvard University.

He was on the Technical Staff of Bolt, Beranek, and Newman, Inc. between 1979 and 1984, during which time he served as resident scientist at the ARPA Acoustic Research Center and the Naval Ocean Systems Center. He has been on the faculty of the Department of Electrical Engineering at Arizona State University since 1989.

Dr. Cochran is a member of the IEEE-SP Conference Board, is an officer in the combined Phoenix chapter of the IEEE Signal Processing and Communications Societies, and is a member of both the American Mathematical Society and the Mathematical Association of America.



Herbert Gish (M'67) was born in Brooklyn, NY, in 1940. He received the B.S. degree in electrical engineering from Syracuse University in 1962 and the M.S. and Ph.D. degrees in applied mathematics from Harvard University in 1963 and 1967, respectively.

He has been with Bolt, Beranek, and Newman, Inc. (BBN), Cambridge, MA, since 1977. His recent research interests have been primarily in the application of statistical pattern recognition methods to the problems of speaker identification and speech recognition. He has also been investigating the statistical properties of neural networks. His previous work at BBN was in underwater signal detection. Prior to joining BBN, he was employed at Signatron, Inc. of Lexington, MA, where his research was in information theory and communications.

Dr. Gish is a member of Eta Kappa Nu and Tau Beta Pi.

Dana Sinno (S'92) was born in Beirut, Lebanon, on August 12, 1969. She received the B.S.E. degree from Arizona State University in 1992.

She is currently a graduate student in the Signal Processing Group at Arizona State University.

Ms. Sinno is a member of Tau Beta Pi, Eta Kappa Nu, and the Golden Key National Honor Society.