Scale Based Coding of Digital Communication Signals

Douglas Cochran and Chao Wei *

Department of Electrical Engineering
Arizona State University
Tempe, Arizona 85287-5706

Abstract— **Multiple access communication schemes using wavelet-based orthogonal codes are described. In the simplest such scheme, the message bit stream in each channel is encoded as a sequence of time-shifted replicates of a mother wavelet symbol at a particular scale. Different channels use the same wavelet symbol but a different scale. The messages are decodable if the time-shifted and dilated replicates of the wavelet symbol used in the encoding are orthogonal. Variations and potential applications of this scale-division multiple access scheme in low probability of exploitation communication systems are discussed. Examples are presented to demonstrate the (Fourier) spectral spreading that can be obtained using this approach.**

I. Introduction

Code-division multiple access (CDMA) protocols are widely used in commercial and military communication systems. They are particularly valuable in settings in which several users must share a communication medium that is cluttered by interference and perhaps intentional jamming.

Implementation of a CDMA system requires development of families of codes which must be at least approximately orthogonal to avoid crosstalk between different users of the system. Traditional frequency-division multiple access (FDMA) schemes rely on the orthogonality of sine waves at different frequencies to define orthogonal "channels" in which users may communicate without interfering with one another. Signals broadcast in a fixed frequency band are particularly susceptible to narrowband interference and are relatively easy to detect and jam, however. Covert, jam-resistant, and robust multiple-access systems must therefore rely on more complicated codes to define orthogonal channels.

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II. Scale-division multiple access

Wavelet theory provides a means for developing rich families of orthogonal codes based on dilations of a single function. Let w be a wavelet and denote

$$w^{(a,b)}(t) = \frac{1}{\sqrt{a}} w\left(\frac{t-b}{a}\right)$$

Consider K bit-stream messages represented by binary functions $m_k : \mathbb{Z}_+ \to \{0,1\}$ for $k = 0, 1, ..., K$. If the collection $w^{(2^k, n2^kT)}$ for $T > 0$, $n \in \mathbb{Z}$, $k \in \mathbb{Z}$ is an orthonormal set in $L^2(\mathbb{R})$, it is apparent that the individual messages m_k can be extracted from the aggregate signal

$$c(t) = \sum_{k=0}^{K} \sum_{n} m_k(n) w^{(2^k, n2^kT)}(t)$$

The recovery mechanism is a wavelet transform on c using wavelet w. In practice, of course, better performance in the presence of noise would be obtained by using a bipolar representation scheme for the bit streams; i.e., by using $+\lambda$ and $-\lambda$ with $\lambda > 0$ rather than 0 and 1 as the values of $m_k(n)$.

Example (Haar wavelet coding): Consider three binary messages

$$\begin{aligned} m_0 &= 1,1,0,1,1,1,0,1,1,1,0,1,1,1,0,1,... \\ m_1 &= 0,1,1,0,1,1,0,1,0,1,1,0,1,1,0,1,... \\ m_2 &= 1,0,0,1,1,1,0,0,1,1,0,1,1,0,1,0,... \end{aligned}$$

These bit streams may be encoded on the Haar wavelet $w_H : \mathbb{R} \to \mathbb{R}$ with

$$w_H(t) = \begin{cases} 1 & \text{if } 0 \le t < T/2 \\ -1 & \text{if } T/2 \le t < T \\ 0 & \text{otherwise} \end{cases}$$

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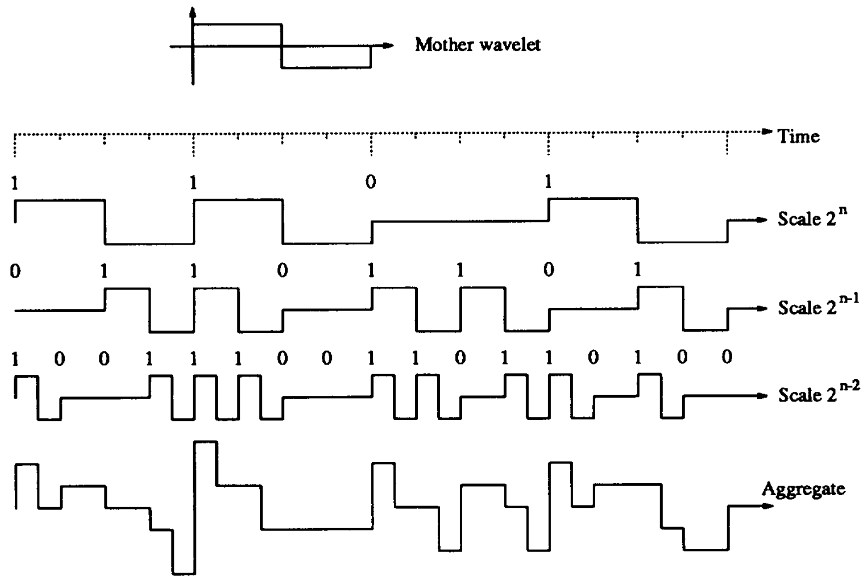

Figure 1: Three bit streams encoded at different scales on the Haar wavelet. Each stream can be recovered from the aggregate signal by a wavelet transform.

as depicted in Figure 1. Note that each of the original messages can be recovered from the aggregate signal by a wavelet transform with wavelet w_H because each of the scaled and shifted replicates of w_H used as a symbol is orthogonal to every other.

Separation of the channels in this multiple access scheme is on the basis of scale, and analogy with similar multiple access schemes with channels separated by frequency, time, *et cetera* suggests the name *scale-division multiple access (SDMA)*. Note that the shifted and dilated replicates of w need not necessarily form an orthogonal basis of L^2 for this multiple access scheme to work. Efficiency will be enhanced if the replicates of w for some practical range of m and n are closely packed in the phase plane, however.

III. Covert and Robust Communications

In the SDMA scheme, various choices of w will give different (Fourier) frequency structures to the aggregate message c and, with knowledge of the statistical behavior of the message sequences m_k, the spectral density of c can be expressed easily in terms of the Fourier transform of w. This feature suggests that SDMA signals could be designed to be robust with respect to narrowband interference or jamming by using wavelets that spread the spectrum of the message sequence. In addition, it may be possible to design SDMA signals that are difficult to detect by traditional means (e.g., radiometers) by choosing w in such a way that the transmitted signals have spectral properties similar to those of ambient noise in the communication environment [3].

Desirable features for wavelets used to spread the spectrum of communication signals in these ways would differ from those used in time-scale analysis of signals. Wavelets compactly supported in time and with *slow* decay in frequency would be well suited to low probability of detection communication coding. Wavelets that distribute the message energy across irregular subbands would provide robustness with respect to narrowband jamming and interference (see Figure 2).

It is also noteworthy that there is a degree of inherent security in SDMA. Although the intended receivers of a broadcast signal (who know w) can recover the individual messages easily, unintended receivers (who do not know w) may have difficulty in doing so. This security could be further enhanced by frequently changing wavelets at times known to the transmitters and intended receivers but not to adversaries (i.e., "wavelet hopping spread spectrum").

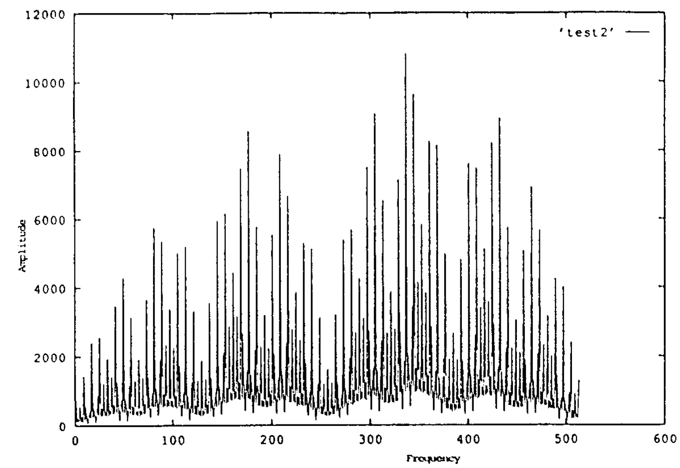

Figure 2: Periodogram spectum estimate of an aggregate signal containing three multiplexed ASCII text messages on a wavelet (not the Haar wavelet). Note the absence of spectral peaks that might be easily detected or jammed.

IV. Channel Capacity and Synchrony

Channels defined by different scales naturally have different capacities measured in bits per unit time. In the dyadic case, for example, the capacity of the channel at scale 2^n is twice that of the channel at scale 2^{n+1}. While this may not be a drawback in some multiple-access applications (where various users need channels of different capacities), the average capacity of each user's channel can be adjusted in at least two ways.

One means of defining channels of equal average capacity is to exploit orthogonality of the time shifted wavelet symbols. Several lower-capacity channels can be obtained by a time-division multiple access (TDMA) protocol within a single high-capacity channel.

Another means of equalizing channel capacity is to assign users rotating sequences of scales for use within specified time intervals. For example, User A might use long (high-scale) symbols in even time slots $[(2k-1)T, 2kT]$ and short (low-scale) symbols in odd time slots $[2kT, (2k+1)T]$ while User B uses short symbols in the even slots and long symbols in the odd slots. In this way, both users would achieve the same average bit rate. While this scheme complicates the communication protocol, it allows the average capacity of each user's channel could be adjusted essentially arbitrarily [4].

The Haar example above suggests that a SDMA system may have to operate synchronously (i.e., with all users having closely synchronized clocks) for the channels to be orthogonal. This is true for the Haar wavelet, but it is not true in general. The following section describes a class of multi-band wavelets that support asynchronous SDMA and also provide spectral spreading.

V. Multi-band Wavelets

As pointed out above, wavelet based codes that distribute the message energy across irregular frequency subbands would be well suited to robust communication in the presence of narrowband interference and jamming. Construction of one class of wavelet codes with this characteristic is possible by extension of a construction of Mallat [2] which is based on a well-known frequency domain orthogonal wavelet basis construction using an ideal bandpass wavelet.

Define a wavelet in the frequency domain by

$$\hat{w}_1(\omega) = \begin{cases} 1 & \text{if } \pi < |\omega| \le 2\pi \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

Then the functions $w_1^{(2^k, n2^k)}$ for $n \in \mathbb{Z}$ and $k \in \mathbb{Z}$ are well known to form an orthogonal basis of $L^2(\mathbb{R})$. Mallat extended this construction, which is based on a wavelet with support in one interval on the positive frequency axis (hence a "one-band" wavelet), to a yield an orthogonal basis based on a "two-band" wavelet. The construction, depicted in figure 3, is based on the wavelet w_2 defined in the frequency domain by

$$\hat{w}_2(\omega) = \begin{cases} 1 & \text{if } 4\pi/7 < |\omega| \le \pi \text{ or } 4\pi < |\omega| \le 32\pi/7 \\ 0 & \text{otherwise} \end{cases}$$

Note that w_2 may be obtained by splitting w_1 into two pieces, the first nonzero only for $\pi < |\omega| \le 8\pi/7$

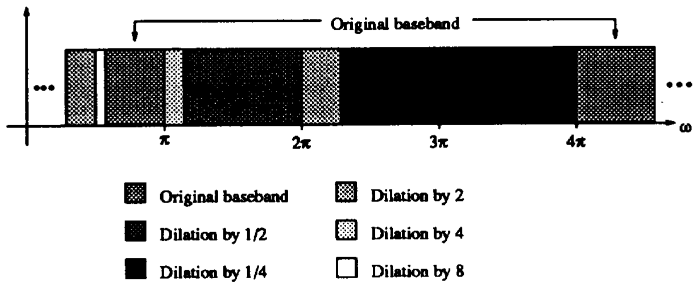

Figure 3: Mallat's "two-band" wavelet basis provides a SDMA code.

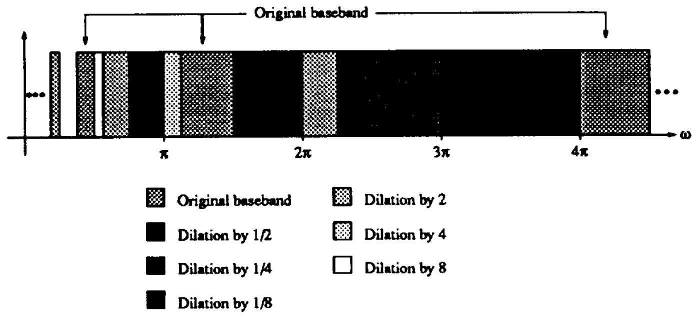

Figure 4: A "three-band" wavelet code for SDMA communications.

and the second nonzero for $8\pi/7 < |\omega| \leq 2\pi$. The first piece is dilated (twice) to four times its original frequency, the second piece is dilated to one half of its original frequency, and they are recombined by addition to yield w_2.

Orthogonal (dyadic) wavelet bases generated by "n-band" wavelets may be constructed by a generalization of this procedure. The original wavelet w_1 is split into n pieces, each piece is dilated by some power of 2, and the pieces are recombined by addition. Figure 4 depicts a three-band wavelet constructed in this way (w_1 is split at $9\pi/8$ and $12\pi/8$; the first piece is dilated to four times the original frequency, the second piece is left in its original band, and the third piece is dilated to 1/4 its original frequency).

The above construction extends easily to provide a fairly rich family of n-band wavelet codes. Note that it applies equally well to generate multi-band orthogonal bases from ***any*** one-band wavelet that yields an orthogonal basis. In particular, there are wavelets other than w_1 on which the constructions can be based, and there is no inherent restriction to dyadic bases. Moreover, because orthogonality is achieved by means of non-overlapping subbands, a SDMA system using these wavelets can operate asynchronously.

Note that these wavelets are not time-limited, however, and truncation to a bounded time interval will result in some crosstalk between the transmission channels. Note that crosstalk due to temporal truncation of bandlimited signals is encountered in numerous widely used communication protocols (e.g., FDMA) without seriously limiting their practical utility.

VI. References

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