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Signal Estimation with a Noisy Saturating Sensor

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SUMMARY[1]

Many biological and man-made sensors *saturate* outside a relatively narrow range of input signal values. Although such sensors are clearly nonlinear, they often exhibit nearly linear response within their range of high sensitivity (Figure 1). The problem considered in this paper is estimation of a signal using noisy measurements from such a sensor.

The sensor has a high-gain (or low noise) central measurement region surrounded by low-gain (or high noise) regions. Estimation schemes discussed rely on the general signal, sensor, and estimator model depicted in Figure 2.

In order to make the best use of the limited high gain sensor range, an estimator can exploit predictability of the input signal. If the high gain region of the sensor is symmetrically centered around zero, this can be done by subtracting the predicted signal value from the actual signal before it enters the nonlinear sensor. This will cause a greater percentage of the measurements from the sensor to be taken in the high-gain region, where the measurement noise has less influence on the measurement. As will be seen, this can greatly improve performance. Shifting of the input signal into the high-gain region of a saturating sensor models an adaptive behavior in biological vision systems known as *subtractive adaptation* [1].

The predictability of the input signal can be modeled by treating it as the output of a linear system driven by white noise. If the sensor response were linear, such a signal would be estimated optimally by a Kalman filter. Thus it is only natural that the estimation methods discussed are Kalman filter variants.

Several estimators were tested in their estimation performance with this system. These estimators are non-adaptive estimators built with knowledge of the system constants and sensor curve.

The first step was to develop a baseline for the comparison of the estimators. It is clear that a lower bound for the performance of a good estimator with the nonlinear sensor is the performance of the Kalman filter with a linear sensor of gain c_{low}, the minimum sensor gain. It is also clear that an upper bound for the performance of an estimator with the nonlinear sensor is the performance of the Kalman filter with a linear sensor of gain c_{high}. These two bounds on the estimators give a region in which performance curves should lie.

[1] An extended version of this paper is available by anonymous ftp from trcsun3.eas.asu.edu.

Sensor Mapping

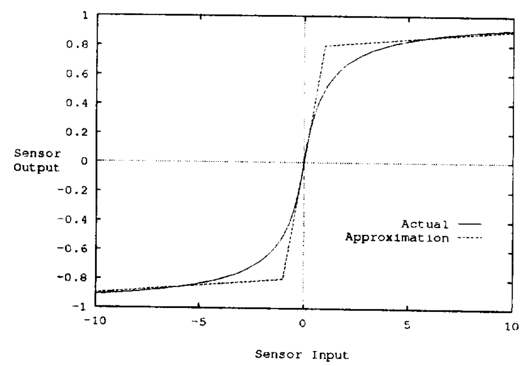

Figure 1: A typical nonlinear input/output relationship for a saturating sensor. Smooth nonlinear sensor gain functions are common in biological and man-made sensory systems; a piecewise linear sensor gain function with high gain c_{high} and low gain c_{low} is used to approximate this relationship in this paper.

The first three estimation methods involve a linear approximation of the sensor. The first of these chooses the sensor gain to be equal to the *equivalent linear gain* of the system. That is, the gain c_γ is chosen so that if the nonlinear sensor is replaced by a linear sensor with gain c_γ, the variance σ_q^2 remains unchanged. Empirical calculation of c_γ requires that there be no prediction; for this reason this estimator turns off prediction by letting $\tilde{s}_k = 0$. The other two linear approximation estimators use prediction and set the gain c to c_{low} and c_{high}, respectively.

The exact inverse estimator uses the exact inverse of the nonlinear sensor to find an approximation of t_k from p_k. This linearizes the sensor at the cost of introducing nonlinear signal-correlated noise. This estimator assumes the noise is Gaussian and uncorrelated to the signal in order to perform its estima-

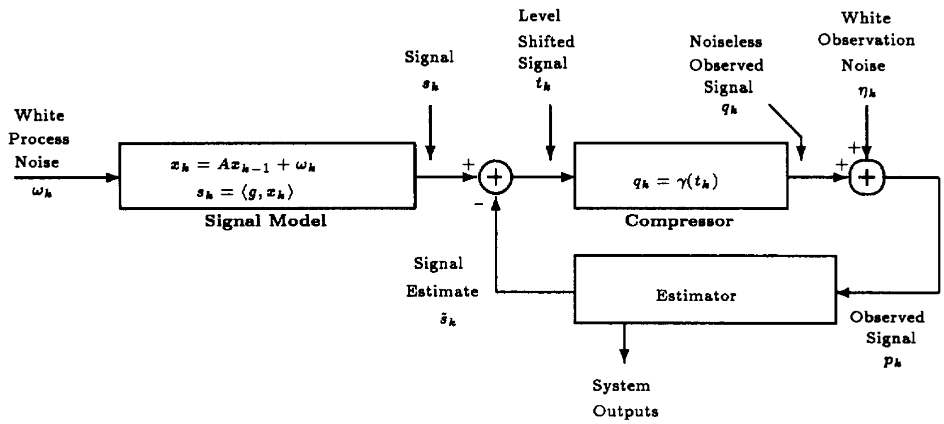

Figure 2: Signal, sensor, and overall system models. The sensor gain function γ is assumed to be invertible and its inverse will generally be incorporated into the estimator. A typical system output from the estimator would be $\hat{s}_k$, the *a posteriori* estimate of s_k.

tion. In performing the inverse, it is necessary to find the variance of the estimate of t_k. This is done in this estimator by a linear approximation of the sensor. Two linear gains were used, c_{low} and c_{high}.

The statistical inverse estimator is similar to the exact inverse estimator, except it finds an approximation of t_k by finding the expected value of t_k given the measured p_k, the distribution of η_k, and the sensor curve. It uses the statistical mean and variance of t_k in its estimates, and hence approximates the noise as Gaussian. Signal-noise amplitude correlation is modeled.

The performance of these estimators can be seen in Figure 3. In this figure, at a measurement noise variance of 0.5 the signals from top to bottom are the high-gain linear approximation, the statistical inverse, the low-gain exact inverse, the equivalent linear gain, the low-gain linear approximation, the lower bound, and the high-gain exact inverse. The upper bound is off the page at this point. On this graph, performance improves as curves tend to the upper right. Hence, the best estimators are the high-gain linear approximation and the statistical inverse. The statistical inverse performs better for low measurement noise and high driving signal variance. But in this region, most of the estimators perform approximately the same. The high-gain linear approximation performs better for low driving signal variance.

The direction for future work is to build an adaptive statistical estimator which will allow estimates of the a-priori distribution of t_k to be used in the inverse of the nonlinear sensor. Presumably this will bring the performance closer to that of the high-gain linear approximation, and possibly achieve an estimator which is optimal in all situations.

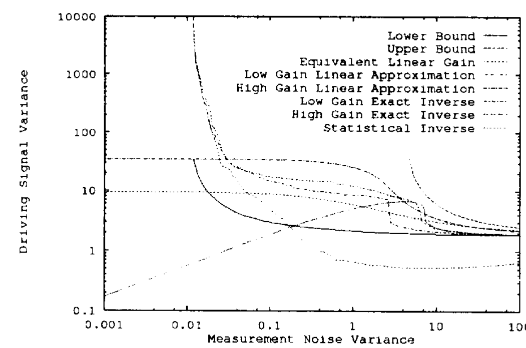

Figure 3: Curves of constant mean-square estimation error obtained from simulations of the various estimators.

ACKNOWLEDGEMENT

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