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A COMPARISON OF NONPARAMETRIC STATISTICS FOR MULTIPLE-CHANNEL DETECTION OF SIGNALS IN OCEAN NOISE

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ABSTRACT

The performance of nonparametric multiple-channel detectors based, respectively, on the multiple coherence estimate and the generalized coherence estimate in detecting broadband signals in simulated deep ocean noise are evaluated are compared. Probabilities of detection estimated from simulations are plotted against probabilities of false alarm predicted theoretically from the distribution functions of the detection statistics under the H_0 assumption that the channels contain independent white noise. The actual false alarm performance for both detection statistics in the presence of simulated ocean noise is seen to be significantly different than what is predicted theoretically for white noise. The common practice of establishing detection thresholds for coherence detectors by using theoretically predicted false alarm probabilities is thus shown to be of limited value when colored noise is involved.

I. INTRODUCTION

An approach for detecting the presence of a common signal on two or more noisy channels based on a test statistic called the Multiple Coherence (MC) estimate was formulated in [6]. This MC detector is nonparametric in that it does not explicitly rely on *a priori* knowledge about the structure of the signal to be detected. Rather, it generalizes a commonly used two-channel detector based on the Magnitude-Squared Coherence (MSC) estimate as a means for measuring the similarity of two arbitrary complex data sequences. Nonparametric detectors are of particular value in applications where it is desirable to detect the presence of a signal, regardless of its structure.

Another nonparametric statistic for measuring the similarity of two or more complex-valued data sequences, called the Generalized Coherence (GC) estimate, was introduced in [5]. It was argued that, since the GC estimate canonically extends the important geometrical structure of the MSC estimate [4], it is a more natural basis than the MC estimate for a generalization of the MSC detector. It was also pointed out in [4] that the the GC estimate has certain mathematical properties that are more desirable in most applications than those of the MC estimate.

Application of the GC estimate as the basis of a nonparametric multiple-channel detector was discussed in [2]. Detection thresholds and preliminary performance data were also presented in that paper.

The goal of this paper is to compare the performance of the MC and GC detectors in detecting a broadband signal in independent channels of simulated ocean noise. Since the GC and MC estimates both reduce to the MSC estimate when only two data sequences are involved, the simplest case in which comparison is meaningful involves three channels. Thus the three-channel case is emphasized in this note. A consequence of the evaluation process was the observation that false alarm probabilities predicted theoretically under the assumption of white noise can be highly inaccurate when dealing with colored noise. Thus, the common practice of relying on such predictions in setting detection thresholds for coherence detectors can yield very unreliable results.

II. THE MSC ESTIMATE

Given a pair of complex data sequences $\mathbf{x}_1 = (x_{1,1}, ..., x_{1,N})^T$ and $\mathbf{x}_2 = (x_{2,1}, ..., x_{2,N})^T$, the *magnitude-squared coherence (MSC) estimate* formed from $\mathbf{x}_1$ and $\mathbf{x}_2$ is

$$\gamma^2(\mathbf{x}_1, \mathbf{x}_2) = \frac{|\langle \mathbf{x}_1, \mathbf{x}_2 \rangle|^2}{||\mathbf{x}_1||^2 ||\mathbf{x}_2||^2}$$

where

$$\langle \mathbf{x}_1, \mathbf{x}_2 \rangle \triangleq \sum_{n=1}^{N} \mathbf{x}_{1,n} \mathbf{x}_{2,n}^*$$

is the inner product of $\mathbf{x}_1$ and $\mathbf{x}_2$, and

$$||\mathbf{x}_i||^2 \triangleq \langle \mathbf{x}_i, \mathbf{x}_i \rangle$$

is the squared magnitude of $\mathbf{x}_i$ for $i = 1, 2$. Note that $\gamma^2(\mathbf{x}_1, \mathbf{x}_2) = 0$ if $\mathbf{x}_1$ and $\mathbf{x}_2$ are orthogonal, $\gamma^2(\mathbf{x}_1, \mathbf{x}_2) = 1$ if $\mathbf{x}_1$ and $\mathbf{x}_2$ are collinear, and $0 \leq \gamma^2(\mathbf{x}_1, \mathbf{x}_2) \leq 1$ for any $\mathbf{x}_1$ and $\mathbf{x}_2$.

In [1], the probability distribution function of the MSC estimate was derived under the assumptions that $\mathbf{x}_1$ and $\mathbf{x}_2$ are independent white gaussian sequences. A geometric argument showing that this distribution remains valid under weaker H_0 assumptions was given in [5]. Knowledge of the statistical behavior of the MSC estimate when no common signal is present on the two channels allows threshold values corresponding to particular false alarm probabilities to be calculated, thereby establishing the utility of the MSC estimate as a detection statistic. In particular, the H_0 distribution of the MSC estimate is given by $Pr\{\gamma^2 \leq r\} = 1 - (1 - r)^{N-1}$ for $0 \leq r \leq 1$.

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III. GENERALIZATION TO SEVERAL CHANNELS

Two statistics that generalize the MSC estimate to several channels have been proposed. The first of these, called the *multiple coherence (MC) estimate*, is described in [6]; the other, called the *generalized coherence (GC) estimate*, is described in [4] and [2].

Three channel MC estimate

When more than two channels are involved, the MC estimate requires that one of the channels be distinguished as a "reference channel." For the purposes of this note, the channels are statistically indistinguishable and the first channel will always be the reference. With this understanding, the three-channel MC estimate is obtained from complex sequences $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ of length N by

$$\gamma^2_{MC}(\mathbf{x}_1 : \mathbf{x}_2, \mathbf{x}_3) = \frac{||\mathbf{x}_3||^2|\langle\mathbf{x}_1,\mathbf{x}_2\rangle|^2 + ||\mathbf{x}_2||^2|\langle\mathbf{x}_1,\mathbf{x}_3\rangle|^2 - 2\mathrm{Re}(\langle\mathbf{x}_1,\mathbf{x}_2\rangle\langle\mathbf{x}_2,\mathbf{x}_3\rangle\langle\mathbf{x}_3,\mathbf{x}_1\rangle)}{||\mathbf{x}_1||^2||\mathbf{x}_2||^2||\mathbf{x}_3||^2 - ||\mathbf{x}_1||^2|\langle\mathbf{x}_2,\mathbf{x}_3\rangle|^2}$$

The H_0 distribution function of the three-channel MC estimate is given in [6] by

$$Pr\{\gamma^2_{MC} \leq r\} = 1 - (N-1)(1-r)^{N-2} + (N-2)(1-r)^{N-1}$$

for $0 \leq r \leq 1$.

Three channel GC estimate

The three-channel GC estimate does not distinguish any particular channel. It is computed by

$$\gamma^2_{GC}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3) = 1 - \frac{g(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)}{||\mathbf{x}_1||^2\mathbf{x}_2||^2||\mathbf{x}_3||^2}$$

where $g(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$ denotes the determinant of the 3×3 *Gram matrix*

$$G(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3) \triangleq \begin{bmatrix} \langle\mathbf{x}_1,\mathbf{x}_1\rangle & \langle\mathbf{x}_1,\mathbf{x}_2\rangle & \langle\mathbf{x}_1,\mathbf{x}_3\rangle \\ \langle\mathbf{x}_2,\mathbf{x}_1\rangle & \langle\mathbf{x}_2,\mathbf{x}_2\rangle & \langle\mathbf{x}_2,\mathbf{x}_3\rangle \\ \langle\mathbf{x}_3,\mathbf{x}_1\rangle & \langle\mathbf{x}_3,\mathbf{x}_2\rangle & \langle\mathbf{x}_3,\mathbf{x}_3\rangle \end{bmatrix}$$

The H_0 distribution of the GC estimate in [2] by

$$Pr\{\gamma^2_{GC} \leq r\} =$$
$$1 - (1-r)^{N-1} - (N-1)(N-2)(1-r)^{N-1}\log(1-r) -$$
$$(N-1)^2[(1-r)^{N-2} - (1-r)^{N-1}]$$

IV. PERFORMANCE EVALUATION

In practice, a common approach for implementing detectors based upon coherence statistics such as the MC or GC estimates is to obtain a detection threshold corresponding to a desired

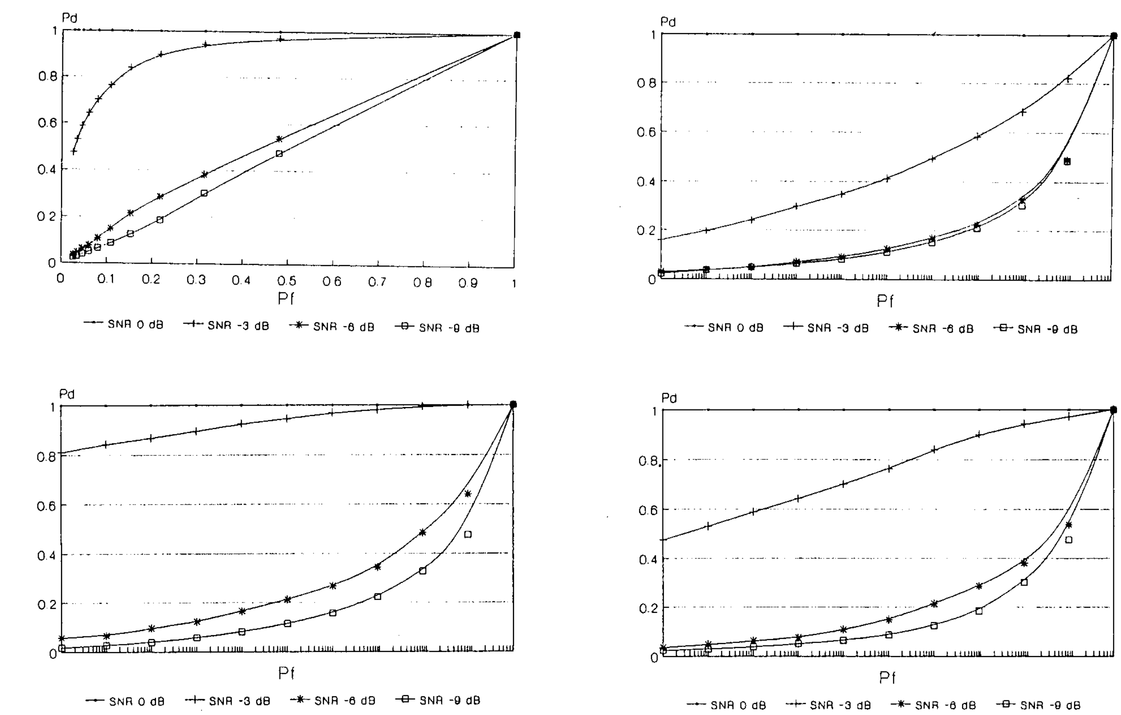

Figure 1. Semilog plots of detection frequency (Pd) versus theoretically predicted false alarm probability (Pf) for a MSC detector with sequence lengths (clockwise from upper left) N =128, 256, 512, and 1024. Signal-to-noise ratios are equal on both data channels.

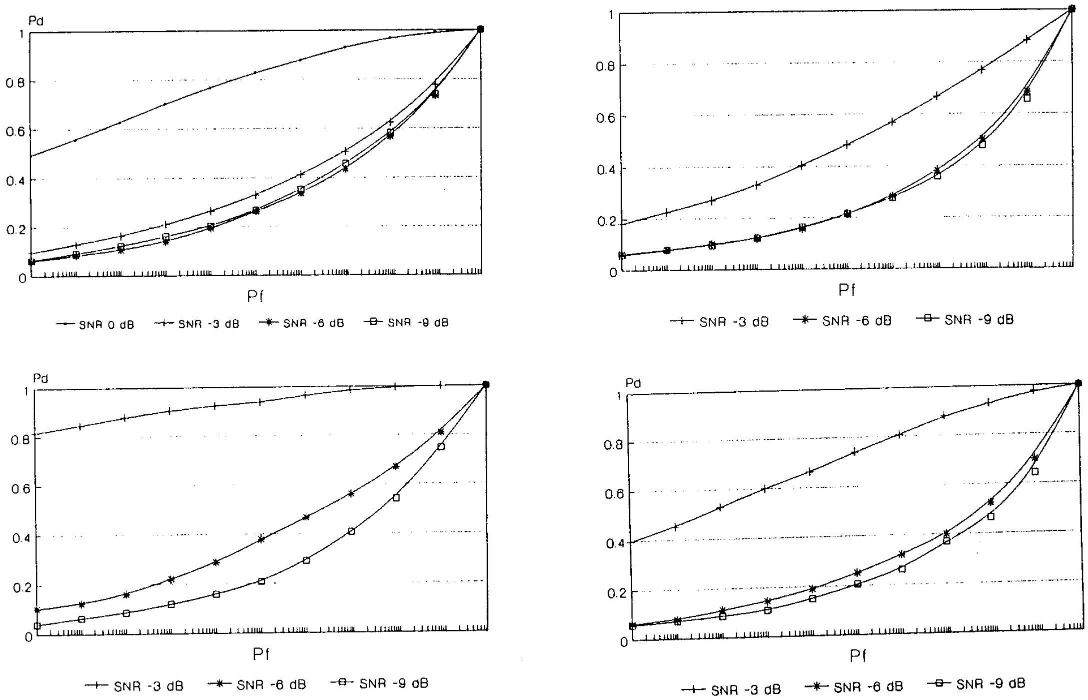

Figure 2. Semilog plots of detection frequency (Pd) versus theoretically predicted false alarm probability (Pf) for a three-channel MC detector with sequence lengths (clockwise from upper left) N = 64, 128, 256, and 512. SNRs are equal on all channels.

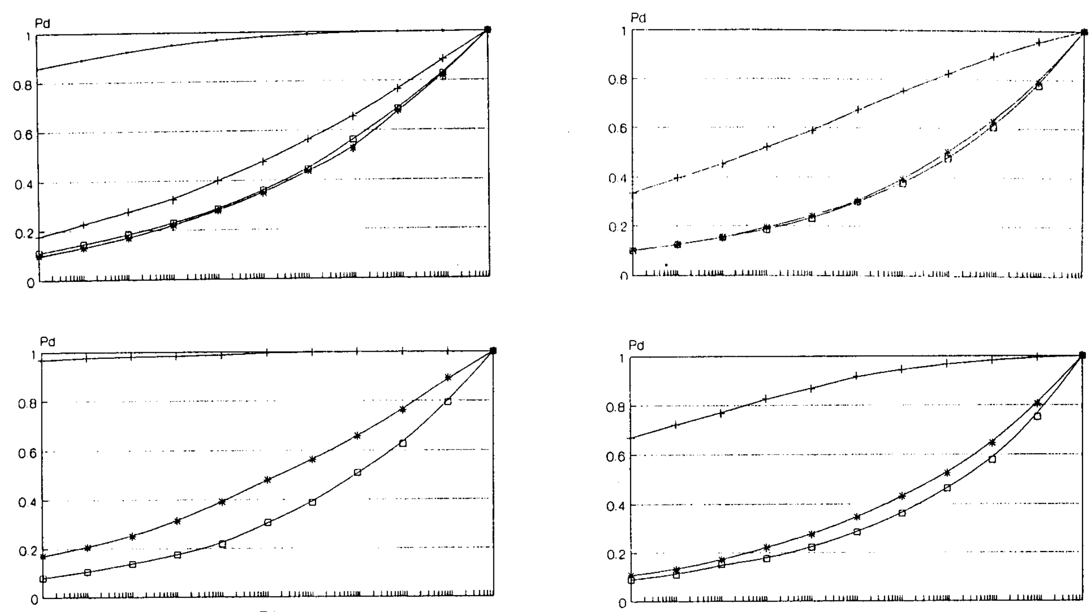

Figure 3. Pd versus Pf for a three-channel GC detector with the same parameters as in Figure 2 above.

probability of false alarm operating point from the H_0 distribution function of the statistic. Although the derivations of the H_0 distributions for the MC and GC estimates contain explicit assumptions about the statistical behavior of the noise, it is often assumed that the false alarm performance obtained in situations where the assumptions are not satisfied can be approximately predicted by these distribution functions.

In this study, a filter was applied to independent white noise sequences to obtain noise sequences with spectral shape similar to deep ocean noise in a band between 10^0 and 10^3 Hertz (see [7], chapter 7). These noise sequences were mixed with a white signal sequence to obtain data with known signal-to-noise ratio (SNR). Between 475 and 2000 (depending on the sequence length N) independent coherence values were computed for various SNR values (chosen to be equal on all channels and in increments of 3 dB) and sequence lengths. Using detection thresholds predicted by the above H_0 distributions to yield a range of false alarm probabilities between 10^{-10} and 10^0, frequencies of detection were associated with each false alarm probability tested.

Probabilities of detection estimated from simulations are plotted against theoretically predicted probabilities of false alarm for a two-channel MSC detector in figure 1 for N = 64, 128, 256, and 512 and SNR values of 0 dB, −3 dB, −6 dB, and −9 dB. Figures 2 and 3 show similar plots for three-channel MC and GC detectors, respectively.

V. DISCUSSION OF RESULTS

In the two-channel (MSC) case and in both three-channel cases, the actual false alarm performance for both detection statistics in the presence of simulated ocean noise is seen to be significantly worse than predicted theoretically for white noise. The common practice of establishing detection thresholds for coherence detectors by using theoretically predicted false alarm probabilities is thus shown to be of limited value when colored noise is involved. Figure 4 shows detection frequencies plotted against actual false alarm rates (note the linear scale) for a MSC detector with N = 512. This figure suggests that an MSC-based detector operating at this bandwidth (1 KHz) in deep ocean noise may be of very limited value. A SNR of −3 dB *in the entire band* appears to be necessary to obtain a 50% detection rate, though longer integration times may help.

With regard to the relative performance of three-channel MC and GC detectors on this environment, this preliminary data indicates that the GC detector is somewhat better. Consider the N = 256 plots in figures 2 and 3, for example. For small detection threshold values (i.e., those corresponding to very low theoretically predicted probabilities of false alarm), the true false alarm rates of the two detectors, as indicated by the −9 dB curves, are fairly close to equal. The detection ability of the GC detector, as indicated by the −3 dB curve, is clearly much better than that of the MC detector.

VI. CONCLUSIONS

The data obtained in this study make one conclusion very evident: prediction of false alarm rates for coherence detectors operating in colored noise environments using probability distribution functions derived under the assumption of white noise is highly inaccurate. This situation might be improved by adjusting the value of N in the distribution function to more accurately reflect the true number of degrees of freedom in the noise. The effective degrees of freedom in the noise might be roughly estimated from bandwidth considerations.

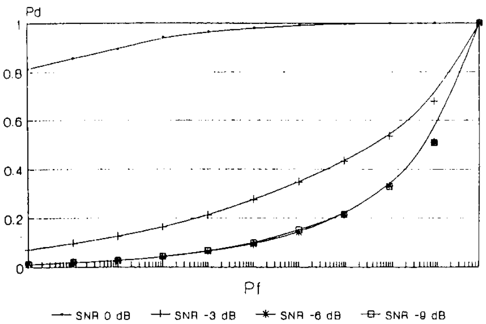

Figure 4. Detection frequency (Pd) versus actual false alarm rate (Pf) for a MSC detector with N = 512. Note the linear scale in Pf.

In order to make a more accurate comparison of the relative performances of MC and GC detectors, a study using white noise will be undertaken.

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